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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

FUTURE OF LAKE MEAD AND ELEPHANT BUTTE RESERVOIR

BY J. C. STEVENS,[1] PRESIDENT, AM. SOC. C. E.

SYNOPSIS

The gates in the by-pass tunnels at Boulder Dam (Arizona-Nevada) were closed February 1, 1935, and the storing of water and sediment began. In ten years following that date some 1,500 million tons of sediment passed the Bright Angel silt-sampling and gaging station in the Grand Canyon, in Arizona. Virtually all of this sediment was arrested by Boulder Dam. There is also an added contribution from the 30,000 sq miles of drainage area between the Grand Canyon Station and Boulder Dam.

Storage in Elephant Butte Reservoir in New Mexico on the Rio Grande began January 6, 1915. Before the construction of the dam a topographic map was made of the reservoir. After its completion five surveys of sediments deposited in the reservoir were made. During this period records of suspended sediments have been kept on the Rio Grande at San Marcial, N. Mex., above the highest backwater from the dam. So far as known this is the only reservoir for which a comparison of sediment inflow and sediments deposited can be made[2] in order to determine some over-all value of the reservoir space occupied by sediment deposits.

An analysis of the sediment records for Elephant Butte Reservoir is presented. Utilizing these data an estimate of the useful lives of both Lake Mead and Elephant Butte Reservoir is made. A discussion of what feasible steps may be taken to prolong their usefulness is included.

Although the writer has served as Consultant for the National Resources Planning Board and also for the Bureau of Reclamation, U. S. Department of the Interior, on matters pertaining to the Colorado River and Rio Grande

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1945.

[1] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[2] "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," *Reports Nos. 1 to 9*, Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943, *No. 9*, p. 19.

basins, the conclusions and opinions here reflect his personal views and are in no sense to be construed as reflecting the opinions of those agencies.

General Considerations

The term sediment, as used herein, will be understood to include all clays, silt, sand, gravel, boulders—in fact, any transported material that will occupy space in a storage reservoir. These sediments are mostly water-borne, although considerable volumes are of aeolian origin, and others are talus from the confining shores of the reservoir. The principal contributions are in the form of deltaic deposits by the main river and by the tributaries that flow directly into the lake.

Deltas.—A delta forms at the mouth of each entering stream. Only deposits at the lowest stage of the reservoir have any chance of remaining in place. All deposits at higher stages are reworked by the streams as the reservoir falls and are thus intermittently carried farther and farther into the reservoir. As these deltaic deposits are reworked they are ground finer and transported farther into the reservoir at every freshet.

The sediments brought in by steep tributaries contain much coarse material. The effect of reworking is to flatten the slope at which they were originally deposited, but such coarse material never moves far into the reservoir.

Density Currents.—The finer sediments, clays, and colloidal material remain in suspension longer than the sands and are therefore deposited well into the reservoir. Frequently such fines are carried by the muddy stream supplying them through the reservoir along its thalweg while the water surrounding this muddy stream remains clear. Such silt flows are called "density currents." By them the fines are deposited against and immediately above the dam. If low outlets are open these mud flows will pass entirely through the reservoir with no visible evidence of their presence on the reservoir surface. By this means the so-called "dead-storage" space lying below outlets may become filled readily with fines.

Talus.—A large quantity of heavy material in the form of talus accumulates from, and settles at, the bases of the steeper shores and cliffs. When exposed above water the talus weathers down into finer sediments but normally does not move far from the base of the cliffs from which it originates.

Aeolian Deposits.—Any one who has driven between Zion National Park, in Utah, and the Grand Canyon can attest to the enormous wind-borne sediments of which many of the bluffs and canyon walls skirting the highway are composed. In arid sections such as these the volume of aeolian deposits is extremely large. Compared with water-borne deposits however their volume is relatively small, yet they cannot be ignored in the long period of years that it takes to fill a reservoir with sediment.

The Life of a Reservoir.—The great complexity in the process of sedimentation in reservoirs and in the inflowing stream channels, the uncertainties regarding the space occupied by sediments, lack of knowledge regarding the

contributions by water and wind from tributary drainage area, the variability of sediment inflows year by year and in wet and dry cycles, the inability to evaluate accurately between suspended sediment and bed load, all mitigate against the drawing of invariable conclusions regarding the useful life of any reservoir that is subject to silting.

PART 1.
ELEPHANT BUTTE RESERVOIR

Elephant Butte Dam was completed in 1916 although impounding of water began on January 6, 1915. The drainage area at Elephant Butte Dam, exclusive of the closed basin in Colorado, is 25,923 sq miles; at the San Marcial gaging station it is 24,176 sq miles. Fig. 1 is a map of the Upper Rio Grande basin on which the alluvium and sedimentary deposits are distinguished from volcanics and metamorphics. The former are largely responsible for the sediments carried by the streams. The Rio Grande is perennially muddy downstream from the mouth of Rio Chama, N. Mex. Upstream from this point it is fairly clear except at times of freshets.

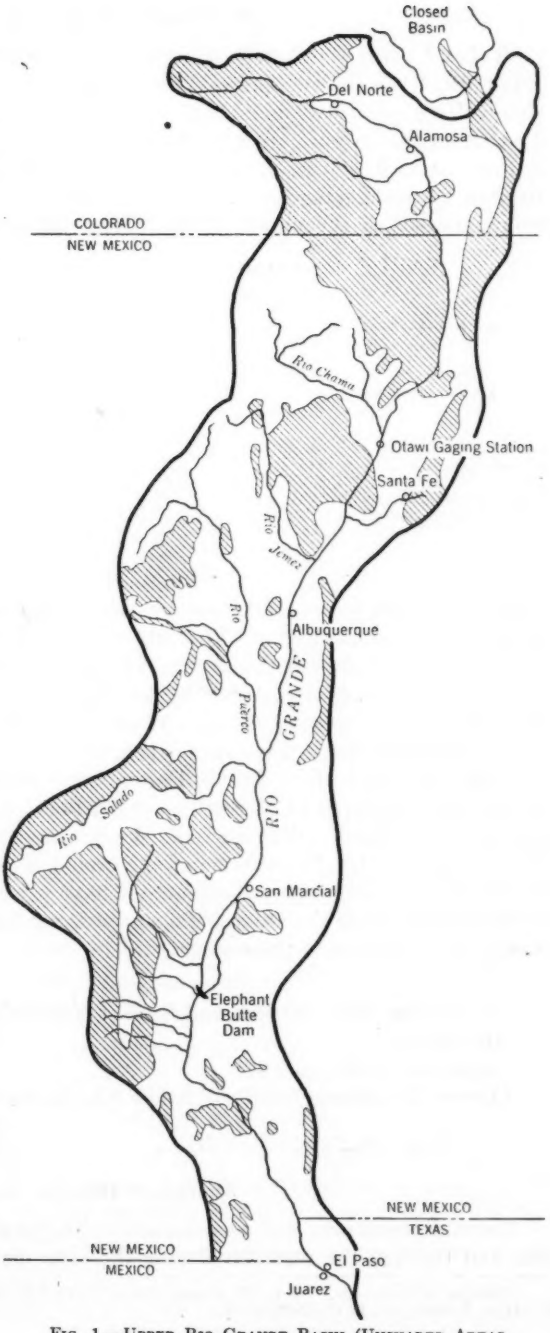

FIG. 1.—UPPER RIO GRANDE BASIN (UNSHADED AREAS ARE SEDIMENTARY DEPOSITS)

Sediments Deposited

Table 1 gives the results of surveys of sediments deposited in Elephant Butte Reservoir below spillway level.[3] It represents the deposits reported from all sources between the sill of the outlet gates and the spillway level. To compare this record with the measured sediments passing the San Marcial gaging station it is necessary to know the sediments deposited downstream from the San Marcial gaging station and above spillway level, as well as the deposits below the sill of the outlet gates. These quantities are not found in any

TABLE 1.—Sediments Deposited in Elephant Butte Reservoir in New Mexico

Period	Years	Cumulative years	Deposited (Thousands of Acre-Feet)		Capacity lost (percentage of original)
			Period	Cumulative	
Storage began January 6, 1915	....	0	0	0	0
January 1915 to December 1916	1.91	1.91	54.0	54.0	2.04
December 1916 to August 1920	3.67	5.58	86.0	140.0	5.40
August 1920 to August 1925	5.00	10.58	91.7	231.7	8.76
August 1925 to April 1935	9.67	20.25	133.5	365.2	13.8
April 1935 to September 1940	5.50	25.75	50.6	415.8	15.8

publications but were ascertained by correspondence to be 13,929 acre-ft and 3,432 acre-ft, respectively. The latter value was changed to 3,218 by a recomputation of capacities made in 1940 (letter to the writer dated January 9, 1943, from Carl B. Brown, Assoc. M. Am. Soc. C. E., Head of the Sedimentation Section, Soil Conservation Service, Washington, D. C.).

For the purposes of this paper it is necessary only to consider the first and last surveys, and to find the total sediment deposited from the beginning of storage in January, 1915, to September 30, 1940—a period of 25.75 years. In the survey of 1935 the deposits in the long arroyos of the west-side tributaries were not included. The volume of the excluded deposits is unknown but for the purposes of this paper 10,000 acre-ft have been added for this item. (An accuracy closer than the nearest 100 acre-ft is not justified by the nature of the surveys.) Following is the summary:

Where deposited	Thousand acre-ft
Above the outlet gate sill and below the spillway level	415.8
In arroyos	10.0
Below the outlet gate sill	3.2
Above the spillway level and below San Marcial gaging station	13.9
Total (19.29×10^9 cu ft)	442.9

Sediment Inflow

These deposits may now be compared with the sediments that have flowed into and through the reservoir during the same period. The records of sus-

[3] "Silting of Reservoirs," by E. M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, p. 12.

pended sediments passing San Marcial begin in 1897, but, since the present study is concerned with the period from 1915 to 1940, only the record for those years is given in Table 2. The data were compiled from the following sources:

TABLE 2.—SUSPENDED SEDIMENTS OF THE RIO GRANDE AT SAN MARCIAL, N. MEX.

Calendar year	No. of samples	Million tons	Calendar year	No. of samples	Million tons	Calendar year	No. of samples	Million tons
1915	....	25.0	1924	....	14.6	1933	59	25.7
1916	....	30.0	1925	....	6.8	1934	44	6.7
1917	....	9.4	1926	32	9.5	1935	66	22.5
1918	....	3.7	1927	28	30.6	1936	66	13.9
1919	....	45.8	1928	17	4.8	1937	322	15.1
1920	....	31.5	1929	49	60.0	1938	365	6.3
1921	....	42.9	1930	8	6.2	1939	365	3.6
1922	....	13.3	1931	41	15.0	1940[a]	274	4.5
1923	....	19.4	1932	45	23.3	Total	....	490.1

[a] Nine months, ending September 30, 1940.

The period from 1915 to 1925, inclusive, was obtained from information supplied by the project engineer, Rio Grande Project, Bureau of Reclamation. The data are based on sampling in 1919 and, for the other years, on monthly silt curves prepared in 1924 from the sampling in 1919 and in years prior to 1913. The data were compiled in acre-feet based on a specific weight of 53 lb per cu ft as used by the late W. W. Follett,[4] M. Am. Soc. C. E. The sediment load was converted to tons by the factor 1 acre-ft = 1,153 tons when $\gamma = 53$ lb per cu ft. The data thus converted were presented by the writer in a former paper.[5] Suspended sediments (in tons) since 1926 are found in the Water Bulletins,[6] published by the International Boundary Commission at El Paso, Tex.

The records for 1915–1925, inclusive, are open to question. Silt curves have been found to be only fair for the coarser sediments but may be quite inaccurate for the fines. However, the errors are compensating and, over a period of years, the totals may not be too far awry. For the years since 1925 the sediment was determined from intermittent sampling throughout each year as shown in Table 2. For some years, notably 1928 and 1930, the number of samples was too few to form the basis of a reliable estimate for the entire year. However, these are all the data in existence for the period in question.

ADJUSTMENTS

In order to compare measured deposits with sediment inflow, certain adjustments in the record of suspended sediments at San Marcial must be made:

1. Samples were secured by lowering small necked bottles "into the water at one or more verticals in the stream section, being careful to approach but not to strike the bottom."[6] The coarser sediment load increases with depth

[4] "Silt in the Rio Grande," by W. W. Follett, Dept. of State, International Boundary Commission, 1913.

[5] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 228.

[6] *Water Bulletins Nos. 1 to 12*, International Boundary Comm., El Paso, Tex., 1931–1942.

until there is a slurry of sands moving along the bottom that is scarcely touched by the sampling bottle.

Fig. 2 is believed to represent, more or less typically, the conditions in a sediment loaded stream. The data are the results of sampling at various depths in ten verticals of the central main canal of the Imperial Irrigation District in the Colorado River delta.[7] The concentration of the coarser sediments is seen to increase rapidly as the stream bed is approached and a reasonably good concentration curve can be drawn. For the fines, however, the concentration is quite uniform from the surface to the bed.

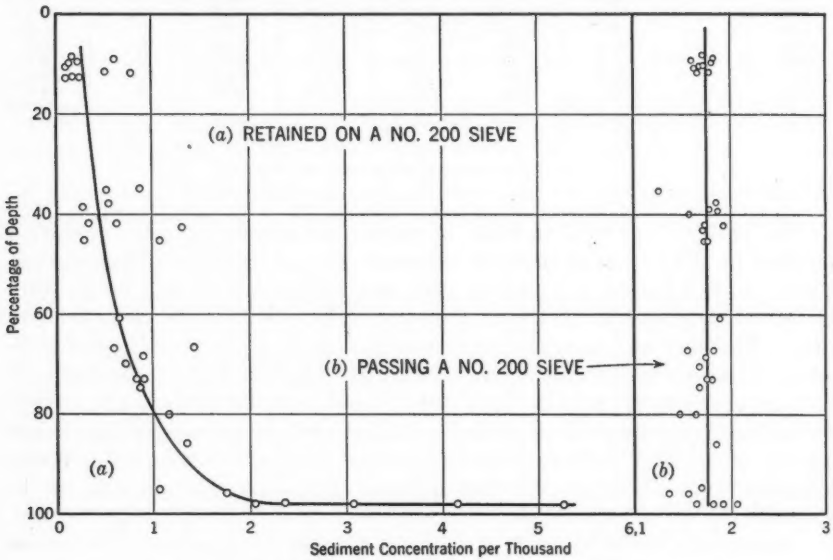

FIG. 2.—SUSPENDED SEDIMENT, IMPERIAL CANAL, CALIFORNIA

To the measured suspended load an addition of 15% was made in order to account for the unsampled bed load. The bed load would thus be 13% of the total sediment load.

2. There are 1,750 sq miles of drainage area tributary to Elephant Butte Reservoir below the San Marcial gaging station. From this arid area many tributaries flow into the lake and substantial wind-borne deposits accumulate in the reservoir (see Fig. 1).

The Soil Conservation Service has made a determination of the sediment contribution by the tributaries of Rio Grande in the valley above San Marcial[8] (see Table 3). This work has been continued since 1939 by the U. S. Engineer Department. Two methods were followed—one by an evaluation of sediment sources through mineralogical studies and the other by sampling suspended loads. The mineralogical studies were made by Gordon Rittenhause for the

[7] "Silt in the Colorado River and Its Relation to Irrigation," by Samuel Fortier and Harry F. Blaney, *Technical Bulletin No. 67*, U.S.D.A., Washington, D. C., p. 47.

[8] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, Tables 11 and 12.

Soil Conservation Service between 1937 and 1941. Composite samples were collected from the bed of the Rio Grande near the mouths of the six principal tributaries and also from farther up each tributary. These samples were sized mechanically. Samples were mounted on microscopic slides for identification and the number of grains in each species was determined. With the specific gravity known, the approximate weight of each species in each size

TABLE 3.—ACRE-FEET OF SEDIMENT PER YEAR CONTRIBUTED BY TRIBUTARIES OF THE RIO GRANDE

Method	Tributary (see Fig. 1)						Specific weights	Sediment (thousand tons)	Drainage areas (acres)	Sediments[a]
	Santa Fe	Galesteo	Jemez	Rio Puerco	Rio Salado	Totals				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Drainage area (sq miles)...	288	697	1,009	6,092	1,434	9,520	....	....	....	....
Mineralogical analyses....	121	362	1,448	4,224	1,569	7,424	86.0	14,400	9,520	1,500
Suspended load sampling..	....	438	684	9,749	769	11,640	62.5	15,800	9,232	1,700

[a] Tons per square mile per year.

grade was computed. The relative abundance of the different mineral species was used as an index of the relative contribution from each of the six tributaries.[9]

Although there are wide differences by the two methods among tributaries, the final values are close enough to justify the use of an average contribution of 1,600 tons per sq mile per yr for the area directly tributary to Elephant Butte Reservoir. The mineralogical analyses purport to show only the deposits on the valley floor; nevertheless they doubtless represent a major portion of the contribution by the tributaries.

3. The talus from the canyon walls that lodges below spillway level should be included. This represents a small volume that is very difficult to determine. Whatever it may be it is considered as included in that of adjustment 2.

4. Some sediment has passed entirely through the reservoir in the form of "density currents."[10] L. R. Fiock[11] estimated that to the end of 1934 a total of 5,000 acre-ft of fine silt had passed through the outlet gates at the dam. This estimate was based on samples some of which contained as much as 6% by weight of fine suspended material.[12] By correspondence it was ascertained that a specific weight of 53 lb was used to determine this volume. The weight, therefore, was 5.8 million tons. It was also learned from correspondence that another major silt flow occurred between August 9 and September 10, 1935, for which the writer estimated that 0.6 million tons of silt passed through the gates. The total passing out during the period of record is therefore 6.4 million tons.

[9] "The Sedimentation in the Middle Rio Grand Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, pp. 145–150.

[10] "The Passage of Turbid Water Through Lake Mead," by Nathan C. Grover and Charles S. Howard, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 720.

[11] "Records of Silt Carried by the Rio Grande and Its Accumulation in Elephant Butte Reservoir," by L. R. Fiock, *Transactions*, Am. Geophysical Union, 1934, p. 468.

[12] *Ibid.*, p. 472.

Specific Weight of Deposits

For the 25.75-yr period from January 1, 1915, to September 30, 1940, the total net sediment inflow to the reservoir becomes:

Item	Source of sediments	Million tons
1	Passing San Marcial	490.1
2	Passing San Marcial as bed load (15% of item 1)	73.5
3	Contributed by 1,750 sq miles of tributary drainage area in 25.75 years at 1,600 tons per sq mile per yr	72.0
4	Total sediment inflow	635.6
5	Less sediment outflow	6.4
6	Net sediment inflow (equivalent to $1,259 \times 10^9$ lb)	629.2

For the same period the measured deposits were 442,900 acre-ft = γ 19.29 $\times 10^9$ lb, from which the average specific weight of the entire volume deposited is $\gamma = \frac{1,259}{19.29} = 65.3$ lb. An average value of 65 lb per cu ft will be used in this paper.

Probable Useful Life

One undertakes a prediction of the useful life of such a reservoir with many misgivings. Nevertheless the best prediction that can be made from available data may be of considerable value even though subsequent years may prove it to have been in error.

The original capacity of Elephant Butte Reservoir was determined from a topographic map on a scale of 1,000 ft per in. with 10-ft contours, made from 1903 to 1908. The volume was recomputed in 1940 and found to be 2,634,800 acre-ft, of which 3,200 acre-ft are below the gate sill.[13]

The cumulative years and the percentage of the original capacity lost, given in Table 1, are shown in Fig. 3. If the average rate of sedimentation shown for the first 26 years should continue and all sediment should remain in the reservoir, it would appear that the reservoir would be full to the spillway level in a total of 158 years, leaving a remaining life of about 120 years more. The useful life, however, would be much shorter because the storage capacity becomes inadequate for the purposes for which it was constructed by the time half its original capacity is lost; at least its usefulness would then be greatly impaired. This would occur in about 80 years from its initial storage date, or about 1995.

These speculations, however, may not hold for the reason that the rate of depositions in the reservoir proper appears to have been decreasing materially as shown by the curved line of loss in Fig. 3. Deposits in the valley above the reservoir have been enormous so that a substantial portion of the water-borne sediments is arrested before it reaches the reservoir. The Soil Conservation Service has estimated that 59,000 acre-ft of sediments were deposited on the

[13] "Silting of Reservoirs," by E. M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524* U.S.D.A., Washington, D. C., 1939, pp. 90–92.

valley floor between Santa Fe Creek and San Marcial in the period from 1936 to 1941,[14] an average of more than 12,000 acre-ft per yr. That Service also reports that aggradation of the lower portion of this valley began eight years prior to the construction of Elephant Butte Reservoir (say, about 1907).

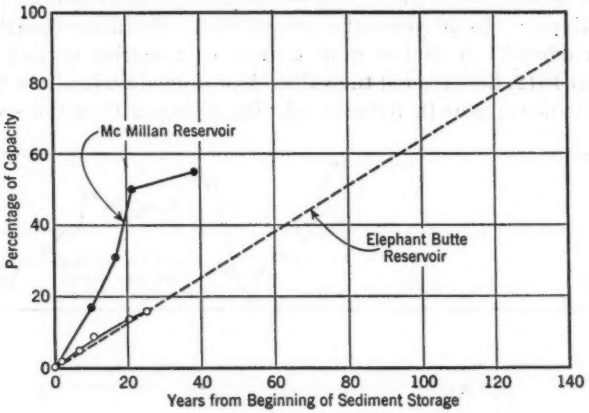

FIG. 3.—COMPARISON OF RATES OF SEDIMENT STORAGE

No very satisfactory explanation has been found for the aggradation of the valley floor. The writer believes it has been caused in part by backwater from the reservoir but mostly by the low runoff cycle the West has been experiencing since 1920. Floods have certain economic values in flushing the river channels of the sediment accumulations of previous low-water periods.

Fig. 4 demonstrates the presence of the low-water cycle. The levels of Great Salt Lake (Fig. 4(*b*))[15] are a good index of cyclic variations in the hydrologic features of the entire West. These levels show the culmination of a high-water cycle in the early 1870's and a continuous decline to 1905, a partial recovery until 1923, and a sharp decline since then.

For comparison, the 5-yr running averages of the flow of the Rio Grande at Otowi Bridge,[16] in percentages of the 52-yr mean, are also shown. The parallelism of the two curves is quite striking for the period of simultaneous records. If the Rio Grande records covered the same period as those of Great Salt Lake, a high-water cycle would undoubtedly have been disclosed between 1865 and 1880.

The Otowi record shows a gradual decline in water yield from the peak of 1909. Slight recoveries appear in both records from the all-time western recorded low of 1934.

If this is a major cause of valley filling, a succession of flood years in a cycle of greater rainfall would very likely move large volumes of the arrested sedi-

[14] "The Sedimentation in the Middle Rio Grande Valley, New Mexico," by Stafford C. Happ, SCS (unpublished), 1942, Table 4.

[15] *Water-Supply Paper No. 880*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1939.

[16] "Regional Planning," Pt. 6 on "The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937," U. S. National Resources Board, Government Printing Office, Washington, D. C.

ments into the reservoir. Although the 25 years since 1920 show declining rates of sedimentation in the reservoir, a series of wet years may increase those rates to such values that the straight-line extension of Elephant Butte curve in Fig. 3 may not be amiss.

In Fig. 3 is also shown the rate of sedimentation of McMillan Reservoir on the Pecos River.[17] In 20 years the rate of filling diminished markedly. The explanation offered[18] is that a rank growth of tamarisk or salt cedar, that started about 1912, has covered the valley floor immediately above the reservoir and that this dense growth reduces velocity so much that the sediments are

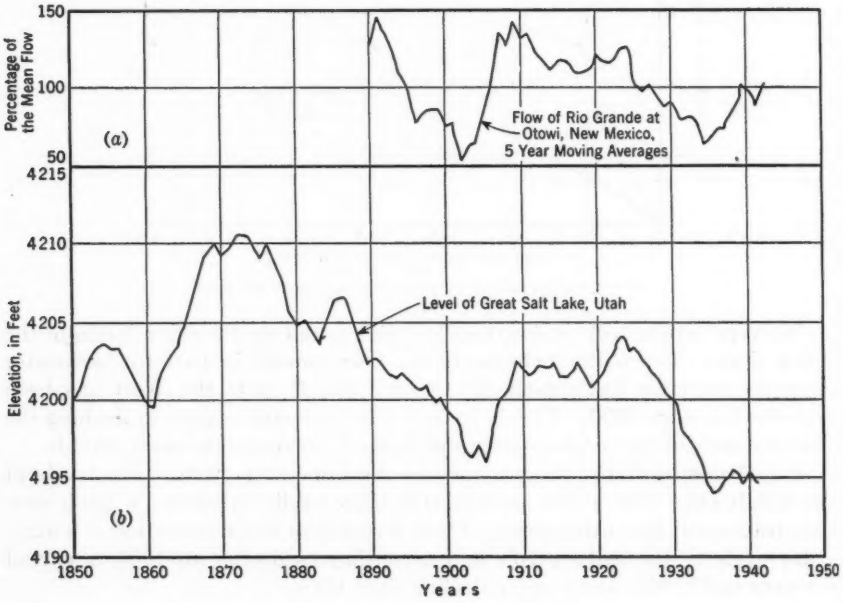

FIG. 4.—STUDY OF HYDROLOGIC CYCLES

deposited in it. No doubt this explanation is correct and as long as the tamarisk thicket remains the sediments deposited will remain in place in spite of years of great floods.

Such extensive growths are not prevalent on the Rio Grande, however; hence the sediments deposited in low-water cycles may very likely be moved out in high-water periods. One of two possibilities appears certain—either aggradation of the Rio Grande above Elephant Butte Reservoir will continue, or the river will again be degraded in subsequent wet cycles which will hasten the filling of the reservoir.

When the survey of Elephant Butte Reservoir was made in 1935 the lake level was so low that practically all the area above the Narrows (somewhat

[17] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 211.

[18] "Silting of Reservoirs," by E. M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, p. 16.

more than half) was mapped by plane table and stadia since the sediments were firm enough to support the men. Therefore, the water for this area had drained out of the upper layers. Without doubt, there was a zone of saturation below the exposed surface of the sediments.

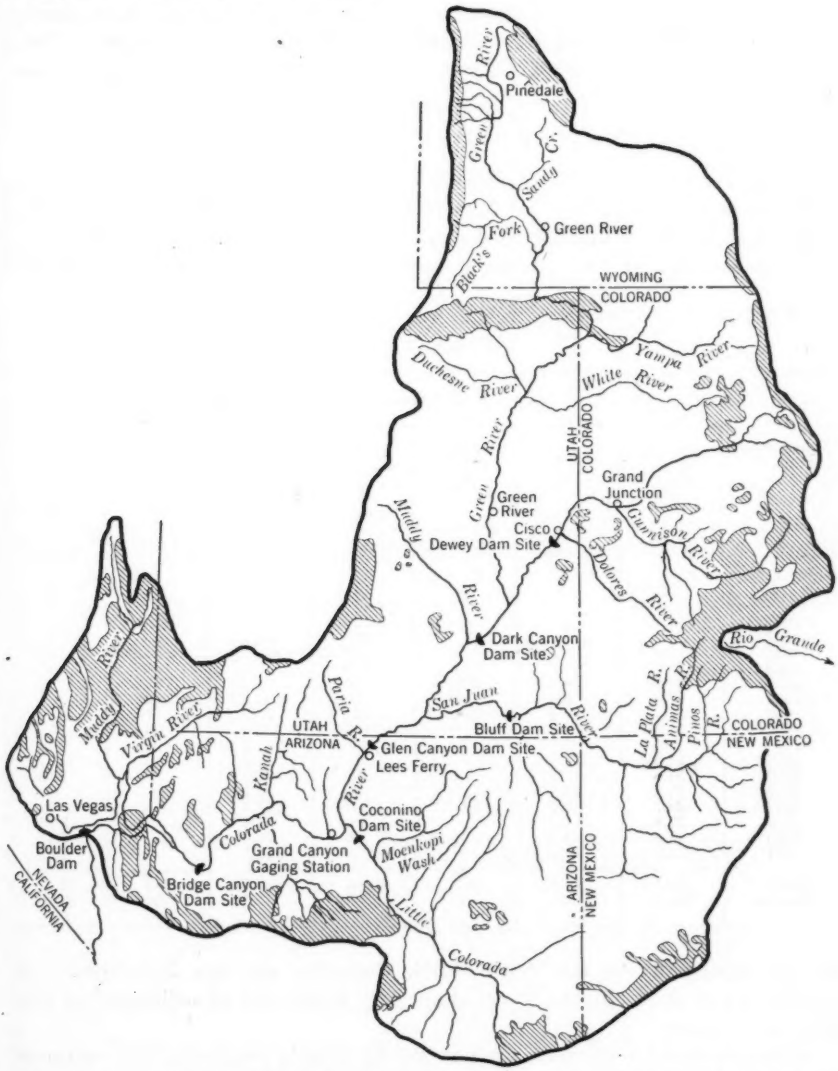

FIG. 5.—UPPER COLORADO RIVER BASIN (UNSHADED AREA REPRESENTS SEDIMENTARY DEPOSITS)

No information is available as to the quantity of gravity water that flowed from the deposits, but whatever it was the loss of storage capacity given should be reduced by that volume. To take account of this ground water with so

many other uncertainties, particularly with regard to valley deposits upstream, seems to be an unnecessary refinement.

A profile of the deposits in Elephant Butte Reservoir[19] shows that reservoir deposits are spread to a considerable extent over the entire bed of the reservoir. There is insufficient evidence to warrant the adoption of other than an average linear rate of filling. The straight line of Fig. 3, therefore, is offered as being a fair index of the useful life of Elephant Butte Reservoir according to present knowledge.

PART II.—LAKE MEAD

Lake Mead was created by Boulder Canyon Dam on the Colorado River, which forms the boundary between Nevada and Arizona in this reach. The gates in the outlet tunnels were closed on February 1, 1935, and the storage of water began on that date. Lake Mead is bordered by an arid plateau which is

TABLE 4.—SUSPENDED SEDIMENTS IN THE COLORADO RIVER BASIN
(Millions of Tons)

Years ending Sept. 30	Location of Sampling Stations (See Fig. 5)						
	Green-river, Utah (1)	Cisco, Utah (2)	Lees Ferry, Ariz. (3)	Grand Canyon, Ariz. (4)	Bluff, Utah (5)	Col. 4 minus Col. 3 (6)	Virgin, Utah (7)
River.....	Green	Colorado	Colorado	Colorado	San Juan	Little Colorado	Virgin
Years of record.....	12	12	5	16	12	5	4
1926	....	....	....	225	....	....	....
1927	....	....	....	398	....	....	....
1928	....	....	....	172	....	....	....
1929	....	....	352	480	....	128	....
1930	34.5	18.6	195	235	58.3	40	....
1931	7.4	9.9	57	69	15.3	12	....
1932	36.1	26.4	214	261	75.3	47	....
1933	15.4	8.2	112	178	23.6	66	....
1934	1.8	2.7	....	50	21.2	....	....
1935	14.3	17.8	....	122	42.7	....	....
1936	33.8	17.2	....	158	32.6	....	3.85
1937	43.4	23.6	....	191	52.2	....	....
1938	38.2	35.7	....	233	52.1	....	....
1939	22.8	14.5	....	86	21.4	....	3.78
1940	8.9	9.9	....	76	26.3	....	2.47
1941	31.9	32.2	....	270	112.7	....	2.32
Total	288.5	216.7	930	3,204	533.7	293	12.4
Average	(24.0)	(18.1)	(186)	(200)	(44.5)	(58.6)	(3.1)

drained principally by the Virgin, Little Colorado, and San Juan rivers. It contributes to the Colorado River about 64% of its load of sediment but only 20% of its water.

Colorado and Green rivers are perennially muddy because of the enormous areas of sedimentary deposits through which they flow. Fig. 5 is a map of the Colorado River basin above Boulder Dam showing the great extent of sedimentary deposits. These consist of shales, friable conglomerates, and sand-

[19] "Silting of Reservoirs," by E. M. Eakin, revised by Carl B. Brown, *Technical Bulletin No. 524*, U.S.D.A., Washington, D. C., 1939, Fig. 19.

stone from the Jurassic to recent alluviums. All streams draining such deposits carry heavy loads of sediment.

Record of Suspended Sediments

Table 4 gives the results of suspended sediment measurements of the major streams of the Colorado River basin above Boulder Dam.[20]

The water carried past each of the sampling stations and the sediment concentration for the period of record are given in Table 5. The results for Little

TABLE 5.—Sediment Concentration, Colorado River Basin, for the Years Given in Table 4

Line	Sampling station	River	Sediment (million tons)	Water Yield		Concentration per 1,000
				Million acre-ft	Million tons	
1	Greenriver, Utah	Green	288.5	67.8	92,400	3.12
2	Cisco, Utah	Colorado	2'6.7	59.3	80,700	2.69
3	Lees Ferry, Ariz.	Colorado	930.0	63.7	86,700	10.7
4	Grand Canyon, Ariz.	Colorado	3,204.0	201.9	274,000	11.7
5	Bluff, Utah	San Juan	533.7	27.4	37,200	14.3
6	....	Little Colorado	293.0	1.8	2,450	119.6
7	Virgin, Utah	Virgin	12.4	0.61	830	14.9

TABLE 6.—Water Yield and Sediment Records (in Millions) Expanded to a 16-Yr Period

No.	Sampling station	Symbol	Period	Water Yield (Acre-Ft)		Sediment (tons)
				Computed	Observed	
	(1)	(2)	(3)	(4)	(5)	(6)
	Colorado River—					
1	Lees Ferry, Ariz.	A_1	1929–1933	63.07	....	930
2	Grand Canyon, Ariz.	B_1	1929–1933	65.5	....	1,223
3	Grand Canyon, Ariz.	C	1926–1941	202.0	....	3,204
4	Lees Ferry, Ariz.; expanded	A_1C/B_1	1926–1941	196.0	195	2,440
5	Cisco, Utah	A_2	1930–1941	59.3	....	217
6	Grand Canyon, Ariz.	B_2	1930–1941	135.0	....	1,928
7	Cisco, Utah; expanded	A_2C/B_2	1926–1941	88.9	90	360
	Green River—					
8	Greenriver, Utah	A_3	1930–1941	42.5	....	288
9	Greenriver, Utah; expanded	A_3C/B_2	1926–1941	63.6	64	479
	San Juan River—					
10	Bluff, Utah	A_4	1930–1941	22.5	....	534
11	Bluff, Utah; expanded	A_4C/B_2	1926–1941	33.7	32	888
	Virgin River—					
12	Virgin, Utah	....	1936–1939 1940–1941	0.61	....	12.4
13	Virgin, Utah; expanded	....	1926–1941	2.66	2.31	48
14	Virgin and Muddy rivers	....	1926–1941	5.0	No record	246

Colorado River (line 6) include contributions from 29,800 sq miles as determined from the difference between the records of Lees Ferry and Grand Canyon, whereas the drainage area of the Little Colorado itself is 24,400 sq miles. The record of Virgin River at Virgin, Utah (line 7), is unpublished. It was obtained by correspondence with the Bureau of Reclamation.

[20] "Suspended Matter in the Colorado River 1925–1941," U. S. Geological Survey, unpublished report.

It will be expedient to expand the records to represent a 16-yr period to conform with that of Grand Canyon. This was done, as shown in Table 6, by using the Grand Canyon record as an index. The method is apparent from the letter symbols in Col. 2, Table 6. As an approximate check on the validity of this expedient, the water yield is expanded as well as the sediment records; then the actual volume of water, as independently measured and reported in Water-Supply Papers of the U. S. Geological Survey, for the 16-yr period is given. The agreement for the water flow is remarkably close for all stations.

The Virgin River and Muddy River expansion (item 14, Table 6) is estimated on the basis of the 4-yr record at Virgin and the 3-yr record of flow of Muddy River at St. Thomas, Nev., from 1913 to 1916 when it was a direct tributary of Virgin River, and taking into account the differences in drainage areas. The expansion for the Virgin and Muddy basins was estimated independently of the Grand Canyon records.

TABLE 7.—SEDIMENT FROM TRIBUTARY DRAINAGE; COLORADO RIVER

Item	Sampling station	Drainage area (sq miles)	Suspended sediment (10^6 tons)	Annual Contribution	
				10^6 tons	Tons per sq mile
	(1)	(2)	(3)	(4)	(5)
	Green River—				
1	Greenriver, Utah	40,600	479	30.0	740
	Colorado River—				
2	Cisco, Utah	24,100	360	22.5	935
3	Item 1 plus item 2	64,700	839	52.5	810
4	Lees Ferry, Ariz.	107,900	2,440	153.0	1,420
5	Item 4 minus item 3	43,200	1,601	100.0	2,310
6	Grand Canyon, Ariz.	137,800	3,204	200.0	1,450
7	Item 6 minus item 4	29,900	764	47.7	1,600
8	Item 6 minus item 3	73,100	2,365	148.0	2,020
	San Juan River—				
9	Bluff, Utah	23,900	888	55.5	2,320
	Virgin River—				
10	Virgin, Utah	934	48	4.0	4,300
11	Virgin and Muddy rivers	11,040	246	15.4	1,400

There are 30,000 sq miles of drainage area between the Grand Canyon sampling station and Boulder Dam, and it is important to know how much sediment this area is likely to contribute. For this purpose Table 7 has been prepared using the expanded records for the 16-yr period from Table 6.

SEDIMENT HELD IN LAKE MEAD

Storage of water and sediment in Lake Mead began February 1, 1935. It is of interest to know the rate at which sediments have been captured by Boulder Dam since closure of the gates. These consist of (*a*) measured suspended matter at Grand Canyon, (*b*) unmeasured bed load, (*c*) sediments contributed by the 30,000 sq miles of intervening drainage area, by both wind and water, and (*d*) talus from canyon walls. From the sum of items (*a*) to (*d*) must be deducted the sediments that passed through the dam in the form of density currents.

(*a*) *Suspended Matter Measured at Grand Canyon.*—The Bureau of Reclamation has prepared a breakdown of the suspended matter measured at Grand Canyon by months since 1935. From all records the following total is obtained:

Dates	Million tons
February 1 to September 30, 1935 (Bureau of Reclamation)	117
October 1, 1935, to September 30, 1941 (Table 4)	1,136
October 1, 1941, to September 30, 1942 (Bureau of Reclamation)	232
Total passing Grand Canyon	1,485

(*b*) *Unmeasured Bed Load.*—The method of sampling is described in a Water-Supply Paper[21] of the U. S. Geological Survey. The U. S. Geological Survey Colorado sampler[22] held a pint milk bottle. A small weight could be released to slide down the line to make a 5/8-in. hole in the paper bottle cap at any desired depth. Usually three samples were taken at each of three verticals at about one fourth, one half, and three fourths the distance across the stream.

An analysis of the accuracy of this sampler as well as of other types is found in a report of the Iowa Institute of Hydraulic Research.[23] For present purposes it is sufficient to state that the greater concentration (particularly of the coarser sediments) is in the 1.5 ft between the mouth of the bottle and the stream bed. For this reason an addition of at least 15% to the measured suspended load for unmeasured bed load is believed to be fully justified.

(*c*) *Sediments Contributed by Intervening Drainage Area.*—Of the 30,000 sq miles of drainage area between Grand Canyon and Boulder Dam 11,040 is in the Virgin and Muddy river basins. Before the filling of Lake Mead, Muddy River was a tributary of the Virgin River, but the lake has now submerged their junction so that each appears as a separate tributary to the lake.

In the 267 miles from the Grand Canyon sampling station to Boulder Dam there are a great many tributaries, the major ones being Bright Angel Creek, Kanab Creek, Havasu Creek, Diamond Creek, Grand Wash, Hualpai Wash, Virgin River, Muddy River, and Los Vegas Wash. All contribute substantial quantities of sediment which in time are discharged directly into Lake Mead or are carried into Lake Mead by way of the Colorado River.

An examination of Table 7 will justify the assumption of an average tributary sediment inflow of 1,600 tons per sq mile per yr, the same as was found for the area tributary to Elephant Butte. This may be conceded to include whatever wind-borne material is carried directly or indirectly into the lake, below the Grand Canyon sampling station.

(*d*) *Talus from Canyon Walls.*—There is no means of estimating the space in the reservoir occupied by the talus material that falls from the canyon walls. The length of the reservoir at high level is about 115 miles, most of which consists of high canyon walls. Since this contribution is a small portion of

[21] *Water-Supply Paper No. 636B*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1929.

[22] "Field Practice and Equipment Used in Sampling Suspended Sediment," *Report No. 1* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1940–1943, p. 149.

[23] *Ibid.*, p. 102.

the total sediment inflow it may be considered as included in the foregoing estimate of tributary inflow.

The Sum of Items (a) to (d).—The silt outflow from Lake Mead occurred after control of the flow began on February 1, 1935, and while the reservoir was filling until the diversion tunnel gates were finally closed May 1, 1936. N. C. Grover, M. Am. Soc. C. E., and Charles S. Howard proposed[10] the following estimate of silt discharged by these turbid outflows:

Silt outflow during:	Million tons
1935	6.0
1936	2.4
Total	8.4

Since closure of the diversion tunnels no turbid waters have passed through the control works. The silt brought in by the density currents is deposited in the lowest portions of the lake bed and against the dam.

Summarizing, the sediment flow during the 7.66-yr period from February 1, 1935, to September 30, 1942, is found as demonstrated by Table 8. A summary

TABLE 8.—SEDIMENT DEPOSITED IN LAKE MEAD (MILLION TONS)

Item	Description	7.66 yr[a]	17 yr[b]
1	Suspended sediments passing Grand Canyon	1,485	3,436
2	Unmeasured bed load at Grand Canyon; 15% of item 1	223	515
3	Contributed by tributary drainage[c]	368	816
4	Total inflow	2,076	4,767
5	Outflow by density currents, 1935 and 1936	8	8
6	Net deposits	2,068	4,759
7	Average annual accumulation	(270)	(280)

[a] February 1, 1935, to September 30, 1942. [b] October 1, 1926, to September 30, 1942. [c] Contributed by 30,000 sq miles of tributary drainage in the years of record stated, at 1,600 tons per yr per sq mile.

similar to Table 8 may be made for the 17-yr period from October 1, 1926, to September 30, 1942, on the assumption that it is representative of future accumulations (see Table 4 and item (*a*)).

SPACE OCCUPIED BY DEPOSITS

For the Lake Mead deposits no better estimate of the specific weight can be made than to use the composite value of $\gamma = 65$ lb found for Elephant Butte Reservoir. On that basis an average annual sediment deposit of 280 million tons per year would occupy a volume equivalent to 198,000 acre-ft.

Fig. 6 gives the storage capacities for Lake Mead. The sediments deposited have remained submerged except the upper portions of the deltas of the main and tributary streams.[24] Only in a cycle of dry years after virtually all water-consuming projects in the upstream parts of the basin have been completed is there likelihood of any substantial portion of the deposits becoming exposed, which would tend to consolidate the deposits.[25]

[24] "Silt Problems," by W. E. Corfitzen, *Civil Engineering*, November, 1942, p. 616.

[25] See *Report No. 9* of "A Study of Methods Used in Measurement and Analysis of Sediment Loads in Streams," Iowa Inst. of Hydr. Research, Iowa City, Iowa, 1943.

Probable Life of Lake Mead

If the foregoing rate of deposition should continue there would be deposited a volume equivalent to the storage capacity at the crest of spillway (28,632,000 acre-ft) in a period of 144 years from the time storage began, or 135 years from 1944.

Sediments are not deposited to uniform depths within the reservoir. Deltas form at the mouth of every tributary and at the inlet of the main stream.

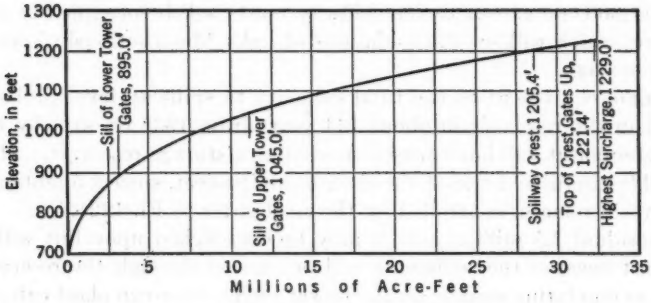

Fig. 6.—Storage Capacity for Lake Mead

These are reworked at varying lake levels; yet they maintain a sloping surface toward the thalweg of the reservoir.[26] Density currents carry fines to the low parts of the reservoir bed and against the dam. The reservoir walls are continually weathering down to form talus slopes at their bases, whereas their upper reaches are being sloped back to stable angles of repose.

The main stream has a tendency to build up its bed above the reservoir to maintain its original slope as it was before its deltaic deposits began to form. If the material is coarse the back sediment curve may extend a long way upstream, as demonstrated by the profile of Bear Creek in California.[27] If the stream joins the reservoir in a flat valley the entire valley floor will be built up to some grade commensurate with the hydrology of the region and the character of the sediments carried, such as is now occurring at the upper ends of Elephant Butte and McMillan reservoirs.

When the river joins the reservoir in a narrow canyon, there is little opportunity for the stream bed to aggrade itself because any deposits left at low and medium stages are carried away in subsequent freshets. Of such is the junction of Colorado River and Lake Mead, since the upper end of the reservoir lies generally in narrow gorges. The opportunity for extensive deposits above lake levels therefore is lacking, and practically all the sediments transported by the river as measured at the Bright Angel station in the Grand Canyon are carried into the lake and cause it to lose storage capacity.

It may be assumed that the climatic conditions over the life span of such a reservoir as Lake Mead will not change materially. The allowances that have

[26] "Sedimentation in Reservoirs," by Berard J. Witzig, *Transactions*, Am. Soc. C. E., Vol. 109 (1944), p. 1057.

[27] "The Silt Problem," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 219.

been made for contributions by tributary areas and bed load do not seem out of place and the storage volume lost by sediment depositions in the lake is based on the best information available.

There are 3,200,000 acre-ft below the lowest gate sill; but water cannot be drawn down to that level. The dead storage, therefore, may be estimated at 4 million acre-ft. At the present rate of sedimentation (198,000 acre-ft per yr), 20 years will be required for the lake to fill the dead-storage space, although encroachment on the active storage has already begun. Half the remaining volume (11,300,000 acre-ft to the spillway crest) will be occupied in about 57 years more, which will see the usefulness of Lake Mead seriously impaired for hold-over storage.

A volume equivalent to the total capacity to spillway level probably will be caught in the reservoir in about 144 years from 1935 or, say, by the year 2080, whereupon it will have ceased to exist as a storage reservoir. However, it will still be useful as a run-of-the-river power project, since 2.5 million acre-ft of storage will remain, controlled by the crest gates to El. 1221.4.

This residual 2.5 million acre-ft may be encroached upon but will not be entirely lost because the sediments will be carried through the reservoir. It will serve as regulating storage for the power such a river-run plant can generate —possibly about half its ultimate installed capacity of 1,300,000 kw.

Prolonging the Life of Lake Mead

Because of the great investment and the importance of this development to the entire southwestern part of the United States, it will be prudent to adopt whatever measures appear feasible to increase the period of usefulness of Lake Mead. The measures to be considered are:

(*a*) Land management on the watershed;
(*b*) Control of density currents;
(*c*) Upstream storage for sediments and other purposes; and
(*d*) Removal of sediments.

Land Management.—The artificial stimulus given to increasing vegetal cover on the watershed, even if it were 100% effective on the areas where it can be practiced, cannot affect the total volume of sediment flow greatly. The areas where such treatment can be used are on range land and forest areas.

If grazing areas are managed to produce their maximum forage yield the erosion prevention feature will have been provided for automatically; that is, the prime objective in management of range land should be to furnish as much food as practicable for live stock on a sustained yield basis.

In areas suitable for forest cover the prime objective should be to produce continuing yields of merchantable timber. The planting and care of forests solely to prevent erosion can rarely be considered a sound financial investment.

In the Colorado River basin, or in fact in any other arid section, natural erosional forces so outweigh those attributable to man's activities that the best that man can do in the way of land management can have only minor effects in reducing the sediments naturally carried by the streams. It is doubtful if

the average of 280 million tons of sediment being carried annually into Lake Mead could be reduced as much as 10% by all the proved practicable methods of land management known today even though lavishly financed.

Density Currents.—Engineers are just beginning to learn something about the stratifications of the contents of reservoirs. Layers of silt-laden waters, as well as those of varying temperatures and of varying concentrations of dissolved solids, remain intact to a remarkable degree. A nontechnical discussion of this subject was published in 1942 by the Department of Agriculture.[28]

Knowledge of such currents was not available, or at least was not recognized to be of possible value, until after Boulder Dam was completed. Facilities for handling density currents and withdrawing water at will throughout the depth of the reservoir are lacking. The technique to take advantage of the current limited knowledge of this subject has not been developed as yet to a practical science. Although it offers some promise in the operation of some future reservoirs under certain favorable conditions, it is not likely to be of much practical benefit for the two reservoirs under discussion. There is also the question as to whether these fines, if deposited on irrigated land, might not have the effect of sealing the interstices of the soil and thus prevent proper air and water drainage.

The subject is worthy of extensive study. Projected reservoirs in arid sections where such density currents and stratifications are likely to exist might well be provided with outlet gates at varying levels in order to take advantage of future possibilities in passing on inflowing sediments.

Upstream Storage for Sediments.—This subject cannot be treated in generalities but will be confined to the specific case of Lake Mead. A number of reservoirs on the Colorado River and silt-laden tributaries have been proposed. The principal ones are Bridge Canyon, Glen Canyon, Dark Canyon, and Dewey Reservoir on the Colorado River; Bluff Reservoir on the San Juan River; and Coconino Reservoir on the Little Colorado. A reservoir for silt storage should also be found on the Virgin and Muddy rivers, but all studies so far made have been concerned with irrigation and flood control, and only incidental studies of the storage possibilities for sediments have been made.

Bridge Canyon Reservoir.—This project has been proposed for additional power. The storage capacity is 2,400,000 acre-ft, of which 1,600,000 acre-ft is active. The sediment inflow to the reservoir may be estimated as follows on the basis of the 17-yr record, 1926–1942.

Item	Description	Million tons
1	Suspended sediments passing Grand Canyon	3,436
2	Unmeasured bed load, 15% of item 1	515
3	Sediments contributed by 7,400 sq miles of tributary drainage in 17 years at 1,600 tons per sq mile per yr	204
4	Total	4,155
5	Average per year	(244)

[28] "Stratified Flow in Reservoirs and Its Use in Prevention of Silting," by Hugh Stevens Bell, *Miscellaneous Publication No. 491*, U.S.D.A., Sedimentation Div., Office of Research, Washington, D. C., 1942.

On the basis of a specific weight, γ, of 65 lb, this would represent an average deposit in the reservoir of 172,000 acre-ft per yr. At this rate the dead storage would be filled in less than 5 years and the reservoir would be completely filled in 14 years. If any power equipment were installed more than that required for a river-run plant, the excess would become virtually useless in about 10 years, which is too short a time for amortization. Any power development therefore should be made with these facts in mind.

Bluff Reservoir.—This reservoir is near the mouth of the San Juan River. The suspended sediments passing this site (Table 6) are 55 million tons per year. The storage capacity has been placed at 1.5 million acre-ft. The sediments from this stream are believed to be coarser than those in the Colorado River. The loss of storage space in the Bluff Reservoir, therefore, may be placed at about 30,000 acre-ft per yr on a basis of $\gamma = 85$ lb requiring 50 years to fill this reservoir. During this period considerable power could be generated but the capacity to be installed should be considered in the light of the possibility that in 30 years the development would revert to a river-run plant.

Coconino Reservoir.—This reservoir would be located in the lower reaches of the Little Colorado River. The proposed storage capacity is 1,600,000 acre-ft. Table 7 shows the difference in the sediment flow between the Colorado River at Lees Ferry and at Grand Canyon to be about 48 million tons per year. Of this quantity 35 million tons may be conceded as coming from the Little Colorado River into this reservoir. The loss of storage space is placed at 23,000 acre-ft per annum on a basis of $\gamma = 70$ lb. Therefore, the reservoir would become filled in 70 years.

This stream would be of no value for a river-run power plant, and it would be unwise to attempt any commercial power development at this site.

Dewey Reservoir.—The dam for this reservoir is proposed on the Colorado River below the mouth of the Dolores River. Its storage capacity as projected is 6 million acre-ft, of which 5 million is active. The sediment passing the Cisco sampling station, from Table 6, was 22.5 million tons per year. If $\gamma = 75$ lb, this sediment inflow would cause the reservoir to lose 14,000 acre-ft of storage space per year. This is a relatively small volume for it would require more than 70 years to exhaust the dead storage alone and 430 years to fill the reservoir.

Glen Canyon Reservoir.—This is perhaps the largest projected artificial reservoir in the world, since it may admit of being developed for a storage capacity of 70 million acre-ft,[29] of which 60 million would be active. The dam would be just above Lees Ferry. It would be 800 ft high and would back water to the foot of the Dewey Dam, a distance of 350 miles.

If the dam were built to this height the reservoir would include the Dark Canyon Reservoir. The foundation and abutment would be in a Jurassic quartzose sandstone of uniform texture that stands in vertical cliffs 1,000 ft or more in height.[30] The crushing strength is from 2,000 to 6,000 lb per sq in.

[29] "Water Power and Flood Control of Colorado River Below Green River, Utah," by E. C. LaRue, *Water-Supply Paper No. 556*, U. S. Geological Survey, Government Printing Office, Washington, D. C., 1925, p. 18.

[30] *Ibid.*, p. 20.

Whether this material would support the loads for a dam 800 ft high or not is a matter for careful consideration.

If Dark Canyon Dam is to be considered, Glen Canyon Dam could only be built to a height of 400 ft and the combined storage of both reservoirs would be only about 14 million acre-ft. Thus, 56 million acre-ft of storage would be sacrificed.

At its highest level the area of the larger lake would be 300,000 acres and at its lowest drawdown 75,000 acres. The annual evaporation from this lake would approximate a million acre-feet per year.

The evaporation from Lake Mead now averages about 800,000 acre-ft per yr. During its later years, on account of increased consumptive use upstream, Lake Mead cannot be held to the high levels possible now, which will result in reducing the annual evaporation over that which obtains at present.

Glen Canyon Reservoir should not be developed to its highest level to run parallel in time with Lake Mead, but should be reserved for a future generation when the usefulness of Lake Mead has become impaired.

Irrigation must be conceded as having a decided priority over power in this region. The loss of 1.75 million acre-ft of water by evaporation, if both reservoirs were in operation simultaneously, is too big a price in water to pay for the temporary advantage in power generation. This loss would irrigate more than a half million acres of land whose value to the nation is far greater than the power that its sacrifice would make possible. Power may be had from other sources but the Colorado River is the only source of water for irrigation in this region.

The sediment flow at the Glen Canyon Dam is that at Lees Ferry (Table 6); namely, 150 million tons per year. The sediments are doubtless somewhat coarser than those at Lake Mead; hence, if $\gamma = 69$ lb, the rate of filling becomes 100,000 acre-ft per yr. This would fill the low-level reservoir of 9 million acre-ft in 90 years. If the high-level reservoir were built it would take 700 years to fill it completely, although its usefulness as a regulator of the flow of the Colorado River would be lost in half that time or less.

The power possibilities at such a high dam and with such enormous storage back of it would be very great indeed. It would probably justify as much as 2,000,000 kw of installed capacity.

Dark Canyon Reservoir.—For the height of dam of 550 ft, this reservoir would have only 5 million acre-ft of storage capacity. It would back the water up the Colorado River to the Dewey Dam and up Green River to the City of Greenriver, Utah. Its normal area would be about 20,000 acres, from which the evaporation would be close to 100,000 acre-ft per yr.

There is some uncertainty as to whether the sedimentary rocks forming the foundations and abutments for the dam would be stable under the loading. If that were decided adversely the stretch would have to be broken up into smaller reservoirs or the project abandoned in favor of the Glen Canyon project.

The sediment that would be caught may be estimated from Table 7, item 5, to be 100 million tons, representing a loss of 61,000 acre-ft per yr on a basis of $\gamma = 75$ lb. This would fill the entire reservoir in 82 years.

Power development at this site would have a useful life of 45 years, after

which the generating capacity would gradually be reduced from that of a storage plant to a river-run plant, but the cost of the equipment sacrificed could be amortized.

A Schedule of Development

In order to determine the effect of sediment storage upstream from Lake Mead a schedule of development was projected from which Table 9 has been prepared. The sedimentation rate is without regard to other upstream reservoirs. The capacity lost is contingent upon the reservoirs upstream.

TABLE 9.—Computation of Storage Capacities Lost in the Reservoirs of the Colorado River Basin (1,000 Acre-Ft Units)

Years (Inclusive)		Number of years	Reservoirs (A = Annual Loss, T = Total Loss, and C = Cumulative Total Loss):												
			Dewey		Glen Canyon		Bluff		Coconino		Bridge Canyon		Lake Mead		
From:	To:		A	T	A	T	A	T	A	T	A	T	A	T	C
River			Colorado		Colorado		San Juan		Little Colorado		Colorado		Colorado		....
Year completed			1950		1970		1960		1960		1955		1935		....
Storage capacity			6,000		9,000		1,500		1,600		2,400		28,632		....
Sedimentation per year			14		100		30		23		172		198		....
1935	1950	16	..	....	..	....	..	....	..	....	..	....	198	3,168	3,168
1951	1955	5	14	70	..	....	..	....	..	....	..	....	184	920	4,088
1956	1960	5	14	70	..	....	..	....	..	....	158	790	26	130	4,218
1961	1970	10	14	140	..	....	30	300	23	230	105	1,050	26	260	4,478
1971	1999	29	14	406	86	2,494	30	870	23	667	19	551	26	754	5,232
2000	2010	11	14	154	86	946	30	330	23	253	0	0	45	495	5,727
2011	2030	20	14	280	86	1,720	0	0	23	460	0	0	75	1,500	7,227
2031	2075	45	14	630	86	3,870	0	0	0	0	0	0	98	4,410	11,637
2076	2167	92	14	1,288	0	0	0	0	0	0	0	0	184	16,928	28,565
Totals		233	..	3,038	..	9,030	..	1,500	..	1,610	..	2,391	..	28,565	....

Obviously many such schedules of development could be forecast, and for each schedule data similar to those of Table 9 could be prepared. The one shown is more illustrative than suggestive of a plan of development. The Boulder Canyon Project Act authorizes the Secretary of the Interior to prepare a comprehensive plan of development for the Colorado River basin. The first draft of that plan has been completed by the Bureau of Reclamation and it is not the intent herein to usurp the prerogatives of that agency in this regard.

In Table 9 the sedimentation periods are recorded for even years only, which explains why the total capacities lost do not check exactly with the original reservoir capacities. The time required to exhaust the entire storage capacity of each reservoir may be found as follows:

Reservoir	Years (inclusive)	Years
Dewey	.....	470
Glen Canyon	1971–2075	105
Bluff	1961–2010	50
Coconino	1961–2030	70
Bridge Canyon	1956–1999	44
Lake Mead	1935–2167	233

Removal of Sediments.—No practicable method of removing the enormous volume of sediments with which this paper deals has been found as yet. Once the sediments have lodged in the reservoir, there they will remain unless some mythical Paul Bunyon and his blue ox can be found to plow them out. Lake Mead, if filled to the crest of the dam, would contain 43 trillion cubical yards of sediment. Even at 1% per cubic yard for removal the cost would be 430 million dollars—about six times the cost of the entire Boulder Canyon Project.

Conclusions

The cumulative totals for Lake Mead from Table 9 are shown graphically in Fig. 7. On the same drawing is shown the capacity lost at the uniform rate of 198,000 acre-ft per yr which it is estimated would occur if the present rate of filling continues. The relatively moderate effect of the upstream storage scheduled on the useful life of Lake Mead is pointedly shown by these graphs. The operation of all upstream reservoirs that could serve to withhold sediments and that now are believed to be feasible will only prolong the total life of Lake Mead 89 years or 62%.

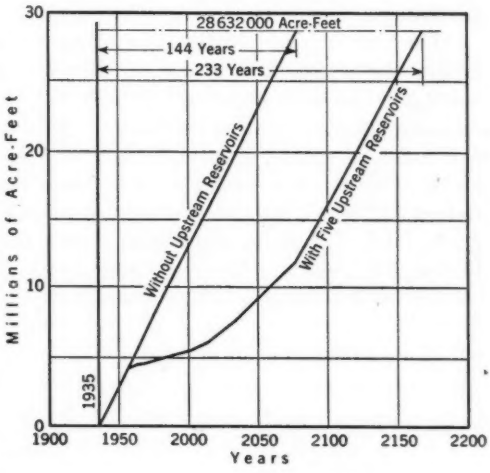

Fig. 7.—Life of Lake Mead

There is a question whether the high dam at Glen Canyon would be found feasible. If it were built, however, it would continue to withhold 100,000 acre-ft of sediment per year from Lake Mead after Bridge Canyon, Bluff, and Coconino reservoirs were filled. This would extend the life of Lake Mead 173 years beyond 2075 to the year 2048—a total life of 314 years from 1935.

If both Lake Mead and Glen Canyon high-level reservoir were operated in parallel, there would be a great loss of water by evaporation. Even if it were pronounced feasible, it is strongly recommended that the construction of Glen Canyon high dam be deferred until some future time when the storage facilities of Lake Mead shall have become seriously impaired.

Bridge Canyon Reservoir would have its life extended from 14 to 44 years by the upstream reservoirs projected, which is still a very short time in which to amortize large investments in power equipment in excess of that suitable for a river-run plant.

In this study Elephant Butte Reservoir was used primarily as an index of the average value of the specific weight of sediments in it, as well as in Lake Mead. The feasibility of constructing reservoirs upstream from Elephant Butte is doubtful. The Rio Grande Compact between Colorado, New Mexico,

and Texas precludes any increase in consumptive use of the water in the Upper Rio Grande basin. Any all-year storage would add the evaporation from them to the present high consumptive use and could well be considered a violation of that compact.

The only course left, therefore, appears to be to continue the use of Elephant Butte Reservoir until its usefulness has become impaired enough to warrant the construction of a substitute reservoir. Even this might increase the present consumptive use. That, however, is a subject for future consideration.

In these studies the effect of land management in withholding sediments permanently from the reservoirs has not been taken into account because its efficacy is so closely bound up with administrative policies and congressional appropriations that its effect, even if known, lacks the essential element of assurance.

The western arid states depend upon irrigation for their very existence. Sedimentation constitutes a direct threat to that continued existence. Something must be done about it, but to do anything engineers must first have data on sedimentation in all of its phases. The available data are pitifully meager, and must be greatly enlarged before any constructive program for relief can be developed. For example, it appears that some 64% of the silt in the Colorado River is contributed by the sedimentary plateau drained by the San Juan, Little Colorado, and Virgin rivers, the area of which is only one tenth that of the Colorado River drainage basin. Here is an area where extensive research and consequent improvements would pay handsome dividends.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

MOVING LOADS ON RESTRAINED BEAMS AND FRAMES

By R. C. Brumfield, M. Am. Soc. C. E.

Synopsis

Engineers have long been familiar with the criterion for placing a load on a simple beam, for maximum moment under one of its concentrations, which requires that the center of the span bisect the distance from the center of gravity of the load to the concentration in question. There is also that other criterion for placing the load for maximum moment at a given point, which requires that the average load in each segment of the span shall be the same as that throughout the span. When the ends of a beam are restrained these criteria cannot be applied; and the problem is further complicated when the object is to find the loading position for maximum moment at the restrained ends. In the past the problem of placing moving loads on fixed spans has usually been solved, rather laboriously, by the use of influence lines.

In this paper the concept of the transmission coefficient will be introduced and equations expressing its value will be derived. From the idea of the transmission coefficient will be developed a method for computing the restraining bending moments at the supports of restrained beams, due to any combination load placed on the span. The resulting equations will be maximized and criteria developed for placing the load in position for maximum bending moment at the support and out in the span. A new concept will be introduced for the sign or sense of the bending moment in rigid frames; and methods will be developed for distributing the moment to the frame in a single operation.

Although the methods contained in this paper may be rendered applicable to beams of all types, the present scope will be limited to straight beams of uniform cross section. Other applications will be outlined but not discussed.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1945.

[1] Associate Prof., Civ. Eng. Dept., Cooper Union, New York, N. Y.

1. Definitions and the Nomenclature of Unusual Terms

As far as practicable, the effort is to conform with standard letter symbols in common use for structural analysis, and these symbols are defined where they first appear in the paper. Inherent in the usefulness of the paper, however, are certain symbolisms peculiar to the method itself, and the nomenclature needs to be understood clearly to grasp the full import of the paper. Such special symbols are as follows:

C = transmission coefficient, a dimensionless fraction by which a bending moment, introduced into one end of a span by forces outside the span, is multiplied in order to obtain the induced bending moment at the opposite end of the same span. This coefficient becomes identical with commonly used "carry-over factor" when its direction of transmission is toward a fixed support.

U = moment of inertia of the loading group, which is equal to the sum of the products of the separate concentrated loads of the group and the squares of their respective distances from the center of gravity of the group.

r = radius of gyration of a series of loads with fixed spacing, found by taking the square root of the quotient that results from dividing the moment of inertia of the group of loads by the total load.

λ = span distribution ratio, $\frac{r}{L}$, or the radius of gyration of the loading group divided by the length of the span on which the load is placed.

S = modulus of dissymmetry, or the algebraic sum of the products of the individual concentrated loads of a group and the cubes of their respective distances from the center of gravity, distances on the right being considered positive and those on the left negative.

e = radius of dissymmetry, or the cube root of the quotient found by dividing the modulus of dissymmetry by the total load.

ϵ = dissymmetry span ratio, $\frac{e}{L}$, the quotient found by dividing the radius of dissymmetry of the loading group by the length of the span in which the load is placed.

i = stiffness index, the product of the span ratio $Q\left(=\frac{L}{I}\right)$ of a member and the quantity $(2 - C)$.

R = restraint index which, for a particular member at a joint, is equal to the reciprocal of the sum of the reciprocals of the stiffness indexes of all other members radiating from the same joint.

m = moment split. For a particular member, moment split is that number by which the moment delivered to the joint by a second member of the same joint is multiplied to obtain the induced moment in the first member.

t = torque split. For a particular member of a joint the torque split is that number by which artificial torque, applied to a joint by some means such as a crank, is multiplied to obtain the induced moment in the particular member to which the split applies.

Q = the quotient found by dividing the span length of a member by the moment of inertia of the span. (It will be noted that this is the reciprocal of the "K-value" often used.)

2. The Transmission Coefficient

The theorem of three moments is familiar to engineers as a tool of computation. It can be derived simply from the use of the second principle of moment areas. When any number of spans radiate from a common rigid joint, the theorem of three moments may be applied to any pair of them, if one member of the pair actuates rotation at the common joint and the other opposes it. Let the two spans being considered be given the numbers 1 and 2. Let the quotients found by dividing the lengths of the spans by their respective moments of inertia $\left(\frac{L}{I} = \frac{1}{K}\right)$ be Q_1 and Q_2. If there are single loads W_1 and W_2 in each of the spans, placed at distances $k_1 L_1$ and $k_2 L_2$, respectively, from the left supports, the theorem of three moments is stated as follows:

$$M_{1L} Q_1 + 2 (M_{1R} Q_1 + M_{2L} Q_2) + M_{2R} Q_2 = - W_1 Q_1 L_1 (k_1 - k^3_1) - W_2 Q_2 L_2 (2 k_2 - 3 k^2_2 + k^3_2) \quad (1)$$

The subscripts L and R refer to the left and right of the spans. When no loads exist in either of the two spans, the right-hand side of the equation becomes zero.

The three contiguous spans, AB, BC, and BD, in Fig. 1, radiate from a common, rigid joint B. The supports at A, B, C, and D are free supports.

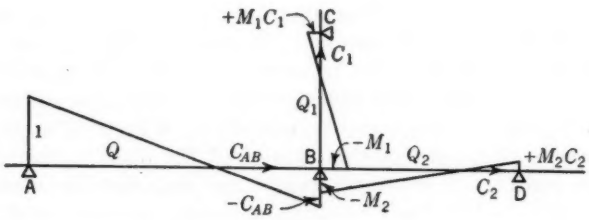


Fig. 1

The spans, as indicated, may continue indefinitely away from the joint B, at supports A, C, and D. Their connections beyond these points are wholly immaterial to this discussion, except as expressed by the transmission coefficient. There are two transmission coefficients C for any span, one in either of the two directions. The letters used to designate the span are used as subscripts to denote the direction of transmission in the span. Thus C_{AB} denotes that coefficient which transmits moment from A to B; C_{BA} that coefficient transmitting moment from B to A.

In Fig. 1, a unit positive couple is assumed to have been introduced into span AB, at end A, from the left. By definition of transmission coefficient, this induces C_{AB} at the opposite end of span AB. The bending moment C_{AB} is negative, according to the ordinary mechanics of materials concept of bending

moment, since the two couples cause a reverse curvature in member AB. The moment $-C_{AB}$ divides at joint B, and induces two unknown moments $-M_1$ and $-M_2$, at joint B, in spans BC and BD. These in turn induce moments of $+M_1 C_1$ and $M_2 C_2$ at the outer ends of their respective spans. Remembering that there are no loads in either of the three spans, the theorem of Eq. 1 may be applied to span AB, in combination with either of the other two; thus:

$$Q - 2\,Q\,C_{AB} - 2\,Q_1\,M_1 + M_1\,Q_1\,C_1 = 0 \quad \ldots\ldots\ldots\ldots (2)$$

from which:

$$-M_1 = \frac{2\,Q\,C_{AB} - Q}{Q_1\,(2 - C_1)} = \frac{2\,Q\,C_{AB} - Q}{i_1} \quad \ldots\ldots\ldots\ldots (3a)$$

and, by analogy,

$$-M_2 = \frac{2\,Q\,C_{AB} - Q}{Q_2\,(2 - C_2)} = \frac{2\,Q\,C_{AB} - Q}{i_2} \quad \ldots\ldots\ldots\ldots (3b)$$

Now the sum of M_1 and M_2 must be equal to C_{AB} in order to have equilibrium at joint B. Adding the values of M_1 and M_2, in Eqs. 3, in terms of C_{AB}, and equating them to C_{AB}:

$$C_{AB} = \frac{Q}{2\,Q + \dfrac{1}{\dfrac{1}{i_1} + \dfrac{1}{i_2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The value of C that appears in the stiffness index i is that one which transmits moment away from the joint of the span, toward which C_{AB} transmits moment.

As defined in Section 1 the reciprocal of the sum of the reciprocals of the stiffness indexes of the receiving members of a joint is known as the restraint index, R, of the transmitting member. If one member of a joint transmits bending moment to that joint, the other members all resist the rotation of the joint due to this moment. The restraint index for this transmitted moment, or rather for the transmitting member, is made up of the reciprocal of the sum of the reciprocals of the stiffness indexes of all the resisting members of the joint. These include all members except the transmitting member. The transmission coefficient C of any particular resisting member, which appears in the stiffness index of that member, will be that one which transmits moment away from the joint under consideration. With these concepts in mind, Eq. 4 can be written:

$$C = \frac{1}{2 + \dfrac{R}{Q}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The Q-value in Eq. 5 is that of the member whose C-value is being computed. The restraint index R contains the entire effect, on C, of all the restraining members, including those directly and remotely connected. When R becomes zero, the value of C becomes 0.5. This is true for a beam with fixed ends. This value of 0.5 is the greatest possible value of C for straight beams of uniform section. Greater values than this may be had when the beam is of non-uniform section. When R becomes infinite in value, the computed value of C

becomes zero. This occurs in the case of a free support. Negative values of the coefficient occur for curved beams.

3. Values of Moment Split

Eq. 5 gives the value of the transmission coefficient for a beam. It is also desirable to know how the moment, which arrives at a joint, such as joint B, Fig. 1, is divided between each of the two receiving members 1 and 2. The multiplier that gives this division is termed the moment split for the receiving member. Such values are readily deduced from the former computation.

Dividing Eq. 3*a* by Eq. 3*b*:

$$M_1 = M_2 \frac{i_2}{i_1} \quad \ldots\ldots (6)$$

Then:

$$M_1 + M_2 = M_2 i_2 \left(\frac{1}{i_1} + \frac{1}{i_2} \right) = M_2 \frac{i_2}{R} \quad \ldots\ldots (7)$$

In the case under consideration, C is equal to $M_1 + M_2$, and

$$M_2 = C_{AB} \frac{R}{i_2} \quad \ldots\ldots (8a)$$

and

$$M_1 = C_{AB} \frac{R}{i_1} \quad \ldots\ldots (8b)$$

in which R is the restraint index for the member, which delivered the moment to the joint, and i is the stiffness index of the (receiving) member whose part of the moment division is being sought. The moment split is the ratio of the two, or:

$$m = \frac{R}{i} \quad \ldots\ldots (9)$$

The sum of these m-values for all the receiving members to which moment divides, from a particular actuating member at the joint, must be equal to unity.

4. Restraining Moments in a Loaded Beam

When the transmission coefficients of a span are known, it is possible to compute, accurately and without the concept of fixed-end moments, the induced restraining moments due to loads placed on the span.

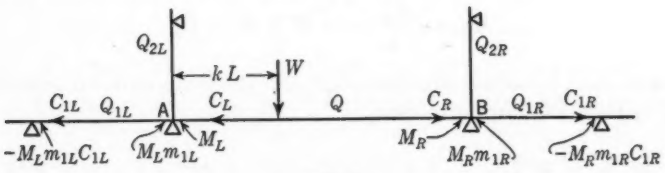


Fig. 2

In Fig. 2, span AB is loaded with a single concentrated load W, placed at a distance $k\,L$ from the left support. The span AB has the two transmission coefficients C_R and C_L, toward the right and left, respectively. Its Q-value is

designated without subscript. The span is shown restrained, at the left and right ends, by two spans at either end. These are designated, in all respects by number-letter subscripts, as 1R—that is, No. 1 span on the right, etc.

Let it be assumed that the induced bending moment due to W, in span AB, at the left end, is M_L, and at its right end is M_R. The induced bending moment in span 1L at joint A will then be $M_L\, m_{1L}$, and at its outer end will be $-M_L\, m_{1L}\, C_{1L}$. A similar set of moments exists in span 1R as shown.

The theorem of three moments, shown in Eq. 1, may now be applied first to the pair of spans AB and 1L, and second to span AB and span 1R. Thus:

$$-M_L\, m_{1L}\, C_{1L}\, Q_{1L} + 2\,(M_L\, m_{1L}\, Q_{1L} + M_L\, Q) + M_R\, Q = -W\, Q\, L\,(2\,k - 3\,k^2 + k^3) \quad \ldots\ldots (10a)$$

and

$$M_L\, Q + 2\,(M_R\, Q + M_R\, Q_{1R}\, m_{1R}) - M_R\, m_{1R}\, C_{1R}\, Q_{1R} = -W\, Q\, L\,(k - k^3) \quad \ldots\ldots (10b)$$

Segregating the M_L-terms and the M_R-terms in each of these equations:

$$M_L \left[\frac{2\,Q + m_{1L}\, Q_{1L}\,(2 - C_{1L})}{Q}\right] + M_R = -W\, L\,(2\,k - 3\,k^2 + k^3) \quad \ldots (11a)$$

and

$$M_L + M_R \left[\frac{2\,Q + m_{1R}\, Q_{1R}\,(2 - C_{1R})}{Q}\right] = -W\, L\,(k - k^3) \quad \ldots (11b)$$

By definition: $Q_{1L}\,(2 - C_{1L}) = i_{1L}$; $Q_{1R}\,(2 - C_{1R}) = i_{1R}$; $m_{1L} = \dfrac{R_L}{i_{1L}}$; and $m_{1R} = \dfrac{R_R}{i_{1R}}$. Therefore, Eqs. 11 become:

$$M_L \left(2 + \frac{R_L}{Q}\right) + M_R = -W\, L\,(2\,k - 3\,k^2 + k^3) \quad \ldots\ldots (12a)$$

and

$$M_L + M_R \left(2 + \frac{R_R}{Q}\right) = -W\, L\,(k - k^3) \quad \ldots\ldots (12b)$$

An inspection of Eq. 5 will show that Eqs. 12 may finally be written:

$$\frac{M_L}{C_L} + M_R = -W\, L\,(2\,k - 3\,k^2 + k^3) \quad \ldots\ldots (13a)$$

and

$$M_L + \frac{M_R}{C_R} = -W\, L\,(k - k^3) \quad \ldots\ldots (13b)$$

Eqs. 13 now may be solved simultaneously for the two unknown quantities M_L and M_R, with the following results:

$$M_L = -W\, L \frac{C_L}{1 - C_R\, C_L}\left[(2\,k - 3\,k^2 + k^3) - C_R\,(k - k^3)\right] \ldots (14a)$$

and

$$M_R = -W\, L \frac{C_R}{1 - C_R\, C_L}\left[(k - k^3) - C_L\,(2\,k - 3\,k^2 + k^3)\right] \ldots (14b)$$

Eqs. 14 give values of the induced bending moments, at the supports of any restrained beam, when a single load W is applied at any point in the span. The negative sign of the bending moment indicates the usual concept of positive and negative bending moment—that is, that the radius of curvature extends from the beam in the same direction as the positive inducing force presses against the beam.

The accuracy of Eqs. 14 may be verified readily. Let the case of a single span be considered, which is fixed at the right end, freely supported at the left end, and is loaded with a single load W at the center. In this case, C_R is 0.5, C_L is 0.0, and k has the value of 0.5. Substituting in Eq. 14*b*:

$$M_R = - W L \frac{0.5}{1 - 0} [(0.5 - 0.5^3) - 0] = - \frac{3}{16} W L$$

Eqs. 14 may be transformed readily into suitable equations for computing the restraining effects of uniform loads. In Fig. 3 is shown a single span with a uniform load of w pounds per foot throughout its length. A single differential portion of this load is indicated at the distance $k L$ from the left. Its value is obviously $w L dk$. This differential load may be considered in the equations as a concentrated load, of which an infinite number make up the total uniform load. Their complete effect may be had by integration, thus:

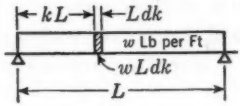


FIG. 3

$$M_L = - w \frac{C_L}{1 - C_R C_L} L^2 \int_0^{1} [(2 k - 3 k^2 + k^3) - C_R (k - k^3)] dk \quad (15)$$

Integrating and substituting the limits, the expression for beams uniformly loaded throughout is:

$$M_L = - \frac{w L^2}{4} \frac{C_L}{1 - C_R C_L} (1 - C_R) \quad (16)$$

Similarly:

$$M_R = - \frac{w L^2}{4} \frac{C_R}{1 - C_R C_L} (1 - C_L) \quad (17)$$

In verification of Eqs. 16 and 17, let the beam previously considered, with a fixed right support and a free support at the left, be loaded with w pounds per foot. Then: $M_R = - \frac{w L^2}{4} 0.5 (1 - 0) = - \frac{w L^2}{8}$.

5. Restraining Moments Due to Combination Loads

Eqs. 14 are entirely serviceable for the case of a span with restrained ends, loaded with a single load. They may be made to serve also for a span loaded with a series of concentrations, by computing the separate effects and adding them algebraically. This is tedious, and therefore it is desirable to modify the equations to apply to the case of a span loaded with a series of loads with fixed spacing such as a truck load, a locomotive load, etc. When so modified the net effect of all loads in the series may be computed at one operation.

In Fig. 4 a single restained span carries two concentrations, W_1 and W_2. The sum of these two loads is W. The center of gravity of the load lies at a distance $\bar{k}\,L$ from the left support. Each of the loads in the series lies at a distance $a_n\,L$ from the center of gravity of the entire group. Thus W_1 is distant $a_1\,L$ from the center of gravity. By definition, the fraction a has the

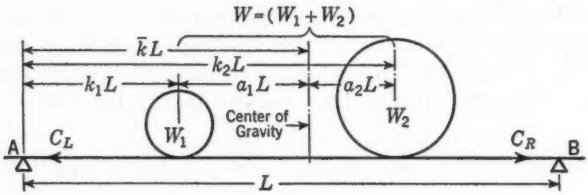


FIG. 4

peculiar property of being negative when the load which it measures lies to the left of the center of gravity, and positive when to the right. All values of a will then reverse sign, when the load is turned around.

For the purposes of substitution in Eqs. 14, it is obvious from Fig. 4 that:

$$k_1 = \bar{k} + a_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and that

$$k_2 = \bar{k} + a_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

also,

$$A_R = \frac{C_R}{1 - C_R\,C_L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Substituting these values of k in Eq. 14*b* and expanding:

$$M_{R1} = -\,L\,A_R\,W_1\,\{\bar{k} + a_1 - \bar{k}^3 - 3\,\bar{k}^2\,a_1 - 3\,\bar{k}\,a^2{}_1 - a^3{}_1$$
$$- C_L\,[2\,(\bar{k} + a_1) - 3\,(\bar{k}^2 + 2\,\bar{k}\,a_1 + a^2{}_1) + \bar{k}^3 + 3\,\bar{k}^2\,a_1 + 3\,\bar{k}\,a^2{}_1 + a^3{}_1]\}\,.\,.(20a)$$

plus

$$M_{R2} = -\,L\,A_R\,W_2\,\{\bar{k} + a_2 - \bar{k}^3 - 3\,\bar{k}^2\,a_2 - 3\,\bar{k}\,a^2{}_2 - a^3{}_2$$
$$- C_L\,[2\,(\bar{k} + a_2) - 3\,(\bar{k}^2 + 2\,\bar{k}\,a_2 + a^2{}_2) + \bar{k}^3 + 3\,\bar{k}^2\,a_2 + 3\,\bar{k}\,a^2{}_2 + a^3{}_2]\}\,.\,.(20b)$$

In Eqs. 20, let $W_1 + W_2$ be equal to W. Adding vertically the like terms of the right-hand side of Eq. 20:

$$M_R = -\,L\,A_R\,\{W\,\bar{k} + (W_1\,a_1 + W_2\,a_2) - W\,\bar{k}^3 - 3\,\bar{k}^2\,(W_1\,a_1 + W_2\,a_2)$$
$$- 3\,\bar{k}\,(w_1\,a^2{}_1 + W_2\,a^2{}_2) - (W_1\,a^3{}_1 + W_2\,a^3{}_2)$$
$$- C_L\,[2\,W\,\bar{k} + 2\,(W_1\,a_1 + W_2\,a_2) - 3\,W\,\bar{k}^2 - 6\,\bar{k}\,(W_1\,a_1 + W_2\,a_2)$$
$$- 3\,(W_1\,a^2{}_1 + W_2\,a^2{}_2) + W\,\bar{k}^3 + 3\,\bar{k}^2\,(W_1\,a_1 + W_2\,a_2)$$
$$+ 3\,\bar{k}\,(W_1\,a^2{}_1 + W_2\,a^2{}_2) + W_1\,a^3{}_1 + W_2\,a^3{}_2]\} \ldots\ldots\ldots (21)$$

In Eq. 21, let: The moment of inertia of the loading group about its own center of gravity equal—

$$\Sigma(W_n\,a^2{}_n\,L^2) = U \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22a)$$

the modulus of dissymmetry of the loading group about its own center of gravity equal—

$$\Sigma(W_n a^3_n L^3) = S \ldots\ldots\ldots\ldots\ldots\ldots (22b)$$

and, by definition of the center of gravity—

$$\Sigma(W_n a_n L) = 0 \ldots\ldots\ldots\ldots\ldots\ldots (22c)$$

Eq. 21 may then be rewritten:

$$M_R = -WLA_R\left[-k^3+k-\frac{3kU}{WL^2}-\frac{S}{WL^3}\right.$$
$$\left.-C_L\left(2k-3k^2-\frac{3U}{WL^2}+k^3+\frac{3kU}{WL^2}+\frac{S}{WL^3}\right)\right]\ldots\ldots (23)$$

Now according to the ordinary definition, the radius of gyration of the loading group is equal to the square root of the quantity U/W. Similarly, the radius of dissymmetry of the loading group may be defined by the equation:

$$e^3 = \frac{S}{W} \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

Making these substitutions in Eq. 23:

$$M_R = -WLA_R\{-k^3+k(1-3\lambda^2)-\epsilon^3$$
$$-C_L[k^3-3k^2+k(2+3\lambda^2)-3\lambda^2+\epsilon^3]\}\ldots\ldots (25a)$$

Similarly:

$$M_L = -WLA_L\{k^3-3k^2+k(2+3\lambda^2)-3\lambda^2+\epsilon^3$$
$$-C_R[-k^3+k(1-3\lambda^2)-\epsilon^3]\}\ldots\ldots (25b)$$

Eqs. 25 are perfectly general and will give the bending moments at the supports of a restrained beam induced by any combination loading except a pure couple.

The equations are entirely usable in the form in which they appear. Their use is greatly simplified, however, by the aid of a curve which obviates the necessity of determining the detailed values of the k and λ-functions for each case. In Fig. 5, the following quantities have been platted for all ordinary values of λ:

$$B = -k^3+k(1-3\lambda^2)\ldots\ldots\ldots\ldots\ldots\ldots (26a)$$

and

$$D = k^3-3k^2+k(2+3\lambda^2)-3\lambda^2\ldots\ldots\ldots\ldots\ldots\ldots (26b)$$

With these substitutions, Eqs. 25 reduce to:

$$M_R = -WLA_R[B-\epsilon^3-C_L(D+\epsilon^3)]\ldots\ldots\ldots\ldots\ldots\ldots (27a)$$

and

$$M_L = -WLA_L[D+\epsilon^3-C_R(B-\epsilon^3)]\ldots\ldots\ldots\ldots\ldots\ldots (27b)$$

in which

$$A_R = \frac{C_R}{1-C_RC_L}\ldots\ldots\ldots\ldots\ldots\ldots (28a)$$

and

$$A_L = \frac{C_L}{1 - C_R C_L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28b)$$

The curves in Fig. 5 are readable accurately to three places, and as a result Eqs. 27 are entirely usable, with satisfactory accuracy, when only slide-rule work is involved.

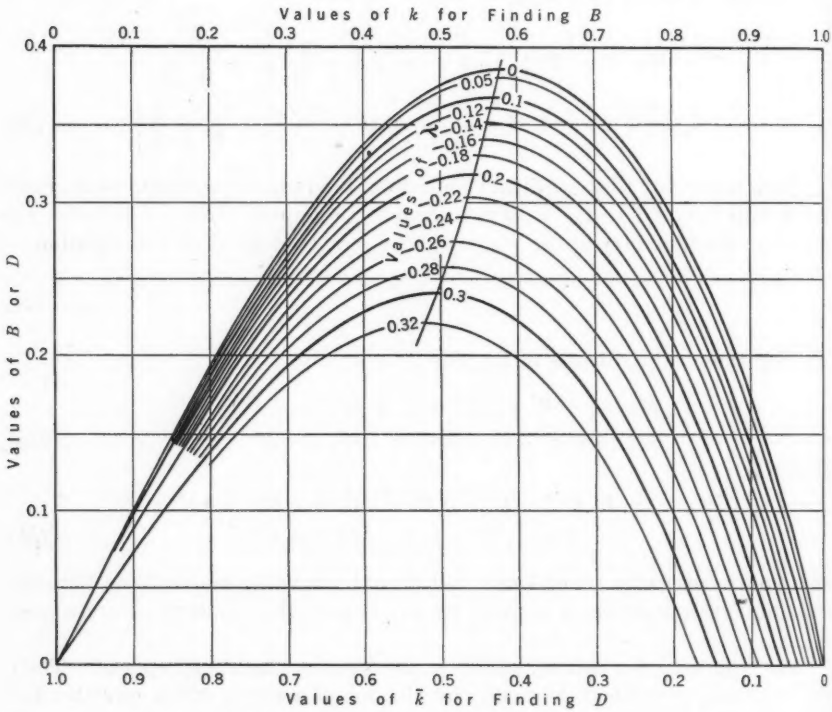


FIG. 5.—CURVES FOR THE SOLUTION OF EQ. 26

As an example of the use of Eqs. 27, let it be required to find the bending moments at the right and left supports of a beam with fixed ends, which is 30 ft long, and which carries a pair of truck concentrations spaced at 12 ft, weighing, respectively, 4 tons and 8 tons. Let the 8-ton concentration be placed at the span center with the 4-ton wheel to the left.

Let the center of gravity be a distance z from the 8-ton wheel; then: $z = \frac{4 \times 12}{12} = 4$ ft; $U = 4 \times 8^2 + 8 \times 4^2 = 256 + 128 = 384$ ton-ft^2; $S = 256 \times (-8) + 128 \times 4 = -2{,}048 + 512 = -1{,}536$ ton-ft^3; $\lambda^2 = \frac{384}{12 \times 900} = 0.0355$; $\lambda = 0.189$; $\epsilon^3 = -\frac{1{,}536}{12 \times 30^3} = -0.0047$; and $\bar{k} = \frac{15 - 4}{30} = 0.367$.

From Fig. 5: $B = 0.276$; and $D = 0.310$. Substituting in Eqs. 27: $M_R = -12 \times 30 \frac{0.5}{1 - 0.5^2} [0.276 + 0.0047 - 0.5 (0.310 - 0.0047)] = -360 \times 0.667 \times 0.1281 = -30.8$ ft-tons; and $M_L = -360 \times 0.667 [0.310 - 0.0047 - 0.5 (0.276 + 0.0047)] = -360 \times 0.667 \times 0.1650 = -39.6$ ft-tons.

6. Maximum Bending Moment Due to Moving Loads

In Eqs. 25 only two variables are involved, when the load is moved on the span, if it retains fixed spacing in movement—the bending moment, and the ratio k. The designer may take a derivative of M with respect to k, equate this to zero, and solve for k, in the resulting quadratic equation. When this is done for each of the two equations, the following two criteria are obtained for placing a load in a restrained span in order to produce maximum bending moment at either of the two restrained supports: For maximum bending moment at the right support—

$$k_R = \frac{C_L}{1 + C_L} \pm \sqrt{-\lambda^2 + \frac{1 - 2C_L}{3(1 + C_L)} + \left(\frac{C_L}{1 + C_L}\right)^2} \quad \ldots\ldots (29a)$$

and, for maximum bending moment at the left support—

$$k_L = \frac{1}{1 + C_R} \pm \sqrt{-\lambda^2 + \frac{C_R - 2}{3(1 + C_R)} + \left(\frac{1}{1 + C_R}\right)^2} \quad \ldots\ldots (29b)$$

Eqs. 29 give the proper locations of the center of gravity of any fixed spacing loading in any restrained span, in order to produce maximum bending moment at the respective supports to which they apply. It is significant that these locations involve only two variables. For maximum moment at the right support the location depends upon the span distribution ratio, and the character of the support at the left end of the beam, as expressed through the coefficient C_L. Similarly, for maximum bending moment at the left support, the location of the load involves only the span distribution ratio and the character of the support at the opposite or right end as expressed through C_R. The dissymmetry of the load is not a factor. The support at the right end may vary in its restraint from fixity to a free support; yet the location of the moving load for maximum bending moment at this support will remain unchanged if the character of the left-hand support is unaltered. Of course, this fact does not prevent the actual maximum from varying in value, as the degree of restraint varies.

Eqs. 29 apply to all kinds of loads, with the exception of a pure couple, which will be treated separately as a special case. Most frequently it is necessary to use the equations in connection with loads, all of which act in the same direction against the beam upon which they are placed. For convenience, the equations have been platted in the form of curves for such cases, in Fig. 6, applying to either of the equations. These curves might well have included only the transmission coefficients from 0 to 0.5, if they were to be used only with beams of uniform section. The curves are also useful in connection with beams of variable section, and for that reason the coefficients represented have been extended to unity.

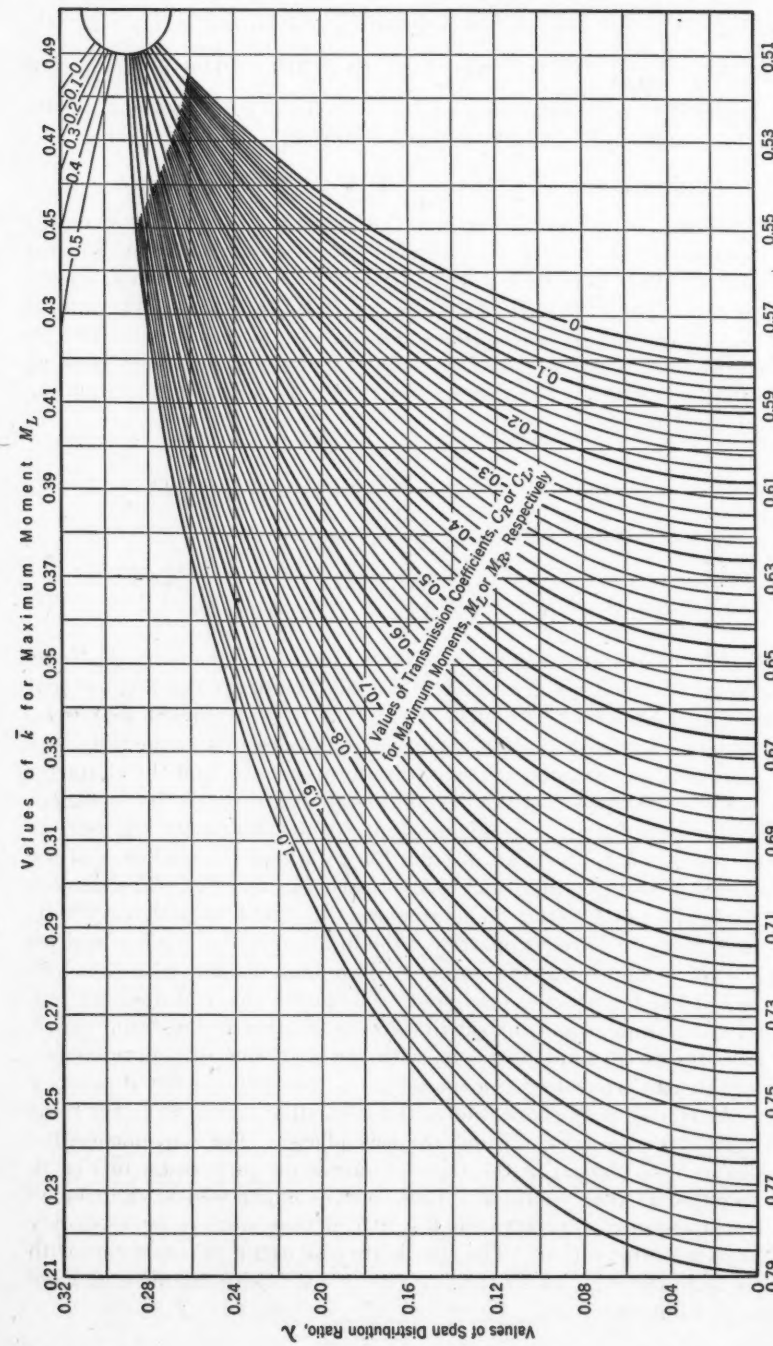

Values of $\bar{k}$ for Maximum Moment M_L
Values of $\bar{k}$ for Maximum Moment M_R
Values of Span Distribution Ratio, λ
Values of Transmission Coefficients, C_R or C_L, for Maximum Moments, M_L or M_R, Respectively
0.21 0.23 0.25 0.27 0.29 0.31 0.33 0.35 0.37 0.39 0.41 0.43 0.45 0.47 0.49
0.32 0.28 0.24 0.20 0.16 0.12 0.08 0.04 0
0.79 0.77 0.75 0.73 0.71 0.69 0.67 0.65 0.63 0.61 0.59 0.57 0.55 0.53 0.51
0.5 0.4 0.3 0.2 0.1 0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0

FIG. 6.—CURVES FOR $\bar{k}$ TO GIVE MAXIMUM BENDING MOMENT, FOR VARIOUS LOADS, AT THE LEFT AND RIGHT SUPPORTS OF ANY SPAN OF ANY RESTRAINED BEAM (FOR VALUES OF λ GREATER THAN 0.289 THE $\bar{k}$-SCALES SHOULD BE INTERCHANGED)

Example.—Let it be required to find the position of the load, in the problem given in Section 5, in order to produce maximum bending moment at the right support, and to compute this bending moment: $\lambda = 0.189$; and $C_L = 0.5$.

Enter Fig. 6 at $\lambda = 0.189$ and follow across horizontally to the value of $C_L = 0.5$, then follow vertically down to the lower horizontal scale, to read: $k_R = 0.6075$. For this value of k and the value of λ given, the designer next finds, from Fig. 5, that $B = 0.318$ and $D = 0.289$. Then $M_R = -12 \times 30 \frac{0.5}{1 - 0.5^2} [0.318 + 0.0047 - 0.5\,(0.289 - 0.0047)] = -360 \times 0.667 \times 0.1805 = -43.3$ ft-tons.

This represents an increase of about 40% over the bending moment obtained with the heavy wheel at the span center.

Fig. 6 represents only a part of the range of results possible with Eqs. 29. When the loads do not all act in the same direction (as would be the case when a column rests on a girder and carries a bending moment at its base in addition to the direct compression in the column), one of the two forces that make up the bending couple acts in a direction opposite to that of the direct compression and the other force composing the couple. In such a case, particularly when the couple is large and the direct compression small, the center of gravity lies outside the loading group, even outside the span in many cases. When such loads occur, λ^2 may be either positive or negative and much greater than represented in Fig. 6. The equations must be solved directly for k in such a case. Any value of k either positive or negative is useful for computing the moment that will cause the loading group to lie entirely on the span.

7. Significance of Span Distribution Ratio and Dissymmetry Span Ratio λ

When loads act in one direction only, the span distribution ratio, λ, becomes a measure of the distribution of the load throughout the span. If λ is small, the load lies bunched near a point. If it is large the load is distributed more generally like a uniform load. In fact, the point at the upper right of Fig. 6 through which all the curves pass represents the value of λ of 0.289 for a uniform load throughout the span. In practice, for loads in one direction only, the value of λ will not exceed this value.

As previously mentioned, the span distribution ratio is a controlling factor in determining the position of the load for maximum bending moment at either of the supports. This is evident from Eqs. 29. A large value of λ will reduce the quantity under the radical in either equation.

As will be shown subsequently, λ is also an important factor in determining the position of a moving load in order to produce maximum positive bending moment under one of the concentrations in the span.

The dissymmetry span ratio, as its name indicates, measures the degree of dissymmetry of the load with respect to its own center of gravity. When perfect symmetry exists the modulus of dissymmetry is always equal to zero. The converse is not always necessarily true—that, when the modulus is zero, perfect symmetry, in the ordinary sense of the word, must necessarily exist.

Fig. 6.—Curves for $\bar{k}$ to Give Maximum Bending Moment, for Various Loads, at the Left and Right Supports of Any Span of Any Restrained Beam (for Values of λ Greater than 0.289 the $\bar{k}$-Scales Should be Interchanged)

In determining the position of the load for maximum bending moment at the supports, the modulus of dissymmetry gives the proper orientation, in order to produce the greatest possible maximum. An examination of Eq. 27*a* will readily show that, for a particular load, M_R is greatest when B and ϵ^3 are additive. This occurs when ϵ^3 is negative. Likewise the greatest possible maximum occurs at the left support when ϵ^3 is positive. The sign of ϵ^3 reverses when the load reverses on the span, although the sign of λ does not change.

It is true that judgment might place the heavy concentration nearest the support at which a maximum is being sought, as the foregoing rules do. Nevertheless, in many cases of mixed loading, the proper orientation is not surely evident. The sign of ϵ^3 is an infallible criterion for the proper orientation.

The value of ϵ^3 is ordinarily small for loads which all act in the same direction against the beam. In cases of bastard couples (parallel unequal forces in opposite directions), however, where the center of gravity falls considerably outside the loading group, the value of ϵ^3 may become quite large and of determining importance in the magnitude of the bending moment.

8. Maximum Positive Bending Moment in Span, at a Given Concentration

Eqs. 27 give the bending moments at the restrained supports, and Eqs. 29 give the proper positions of the load to produce maximums. It is also often desirable to locate the load properly, so that it will produce a maximum positive bending moment under a given concentration out in the span. The designer is interested only in the moment under a concentration, since maximum bending moment occurs at a point of zero shear and shear may only change, in a straight beam, at a load concentration.

In Fig. 7(*a*), a span of length L is loaded with three fixed spacing concentrations W_1, W_2, and W_3, whose center of gravity lies at a distance $\bar{k}$ from the left support. It is required to place the combination load in the proper position for maximum bending moment under the concentration W_2. The simple beam bending moment diagram is shown in Fig. 7(*b*), with the moment under W_2 indicated as $W L (1 - \bar{k}) (\bar{k} + a_2) -$ a constant. This represents the simple beam left reaction, times the distance to W_2, minus the constant, which is W_1 times the distance from W_1 to W_2. The separate complete bending moment diagrams, produced on the simple beam, by the two restraining couples at the ends, are represented in Figs. 7(*c*) and 7(*d*) by the two triangles, whose vertical legs are labeled M_L and M_R. The bending moment produced by each of these restraining couples

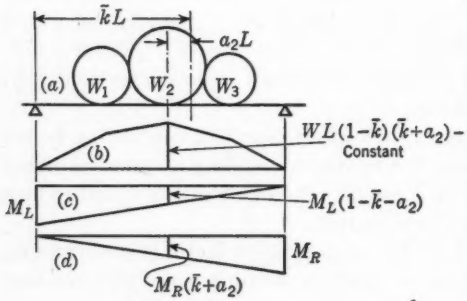


Fig. 7

under W_2 is marked on the respective triangles as $M_L\ (1 - \bar{k} - a_2)$ in the case of the diagram for M_L, and as $M_R\ (\bar{k} + a_2)$ for M_R.

The net bending moment under W_2 will now be the sum of the three separate ordinates, marked on the three bending moment diagrams. It is given by the following equation:

$$M_{W2} = W\,L\,(1 - \bar{k})\,(\bar{k} + a_2) - \text{a constant} + M_L\,(1 - \bar{k} - a_2) + M_R\,(\bar{k} + a_2) \quad \ldots\ldots (30)$$

There are potentially only two variables in Eq. 30, when the load is moved. These are (1) the bending moments under W_2 and (2) the value of $\bar{k}$. Taking the first derivative of M_{W2} with respect to $\bar{k}$, equate this expression to zero, and solve for $\bar{k}$, thus locating the position of the load for maximum bending moment under W_2:

$$\frac{dM_{W2}}{d\bar{k}} = W\,L\,(1 - 2\,\bar{k} - a_2) + (M_R - M_L) + \frac{dM_R}{d\bar{k}}\,(\bar{k} + a_2) + \frac{dM_L}{d\bar{k}}\,(1 - \bar{k} - a_2) \quad \ldots\ldots (31)$$

Eq. 31 may be set equal to zero after proper substitutions are made for the various quantities (which have been only indicated in this paper). Thus the values of M_R and M_L are the right-hand sides of Eqs. 25*a* and 25*b*, respectively. The values of $dM_R/d\bar{k}$ and of $dM_L/d\bar{k}$ are determined by taking the derivatives of Eqs. 25, and these values are substituted in Eq. 31. The process is not difficult but the algebra is tedious. It will not be given herein, but the result will be indicated.

The following substitutions can be made for combinations of factors:

$$q_1 = (C_R + 2\,C_R\,C_L + C_L) \quad \ldots\ldots (32a)$$

$$q_2 = (C_L + C_R\,C_L) \quad \ldots\ldots (32b)$$

$$f_3 = 4\,q_1 \quad \ldots\ldots (33a)$$

$$f_2 = 3\,(q_1\,a - 4\,q_2) \quad \ldots\ldots (33b)$$

$$f = 6\,q_1\,\lambda^2 + q_2\,(4 - 6\,a) + 2\,(3\,C_L - C_R - 1) \quad \ldots\ldots (33c)$$

and

$$\kappa = 2\,C_L - 1 + a\,(1 + C_R) + q_2\,(6\,\lambda^2 - 2\,a) - q_1\,(3\,a\,\lambda^2 + \epsilon^3) \quad \ldots (34)$$

It is to be noted that the symbol a in Eqs. 33 and 34 denotes that value which applies to the particular load under which a maximum bending moment is to be sought. This is a_2 in the case under discussion. When Eq. 31 is equated to zero, with the indicated substitutions, it reduces to:

$$f_3\,\bar{k}^3 + f_2\,\bar{k}^2 + f\,\bar{k} = \kappa \quad \ldots\ldots (35)$$

When the values of C_R and C_L each become zero, reducing the case to a simple beam, Eq. 35 reduces to the ordinary criterion for placing a load on a simple beam for maximum bending moment under a given concentration.

Because of limitations of space no example of the use of Eq. 35 is offered; but its solution is not difficult if a table of squares and cubes is used in con-

nection with a cut-and-try method of solution. It is sufficient to state that Eq. 35 gives results very little different from those of the ordinary criterion for simple beams, in cases where the values of C_R and C_L are equal, or approximately equal. In such cases the criterion for simple beams may be used for the load placing, with satisfactory accuracy.

Even in cases where there is a decided difference in the restraints at the two ends of the beam, the simple beam criterion is useful for a preliminary estimate of load position. If it is remembered that the criterion of Eq. 35 will always place the load somewhat nearer the end of low restraint than does the simple beam criterion, and if such a shift is made arbitrarily according to acquired judgment, it is possible in most cases to avoid the use of the more complicated criterion of Eq. 35.

The criterion of Eq. 35 gives the position of the load for maximum bending moment under a particular load, where this moment is due to the moving load only, and is not a combination with any dead load that may exist on the span. It is easily possible to extend the foregoing procedure to the case of the absolute maximum produced by combining live-load and dead-load bending moments. It is usually sufficiently accurate and safe to add the maximums due to the two causes as separately obtained.

After the load is placed, it is desirable to find the actual bending moment under the wheel, by first using Eqs. 27 to find the restraining moments at the supports, and then determining the positive bending moments from these, using the laws of equilibrium.

9. Bending Moments at Supports of a Restrained Beam Due to a Pure Couple

Eqs. 27, for the restraining bending moments at the supports of a loaded span, do not apply in the case of a pure couple. The center of gravity of a pure couple lies at an infinite distance to one side of the center point between the forces that comprise the couple. As a result the moment of inertia and the modulus of dissymmetry of the load about its own center of gravity become meaningless. The span distribution ratio and the dissymmetry span ratio also lose their meaning. Since these values may not be computed they cannot be substituted in the equations and these are therefore rendered valueless for computing the restraining moments, resulting from a couple on the span.

Eqs. 14, however, may be modified in such a manner as to be valid for a couple. Fig. 8 shows a clockwise couple, composed of two equal forces W, acting in opposite directions against a span of length L, and spaced at a distance $2\,b\,L$ apart. The center point between forces is placed at a distance $k\,L$ from the left support. The load nearest to a support, $-W$, is at a distance $(k-b)\,L$ from the left support, and the farther load is at a distance $(k+b)\,L$. The value $(k-b)$ may now be substituted for k in Eq. 14*a*. The resulting equation will give the bending effect of the left-hand load in terms of the location of the center point between couples. The value $k+b$ may next be substituted in the same equa-

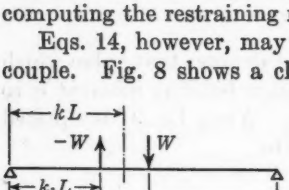


Fig. 8

tion with a second equation resulting which gives the independent effect of the right-hand load, in inducing bending moment at the left support.

If the first equation is now subtracted from the second (since the effect of the left-hand load is opposite to that of the right-hand load), the resulting equation will give the net effect of the couple in inducing bending moment at the left support.

When these steps are taken, remembering that the value T of the couple is $T = 2\,W\,b\,L$, there results:

$$M_L = -\,T\,A_L\,[2 - 6\,k + 3\,k^2 - C_R\,(1 - 3\,k^2) + b^2\,(1 + C_R)] \ldots (36a)$$

and

$$M_R = -\,T\,A_R\,[1 - 3\,k^2 - C_L\,(2 - 6\,k + 3\,k^2) - b^2\,(1 + C_L)] \ldots (36b)$$

The two equations, Eqs. 36*a* and 36*b*, give the restraining moments at the two supports of any restrained beam loaded with a couple placed at any point in the span. A negative sign of the bending moment here means the same thing it has meant in all the other equations for restraining moment—that the radius of curvature of the beam as bent at the support extends from the beam in the same direction as that force of the couple, which is considered positive, presses against the beam. The couple T is considered positive when it is clockwise from the viewpoint of the observer. Otherwise it is negative.

As a practical matter, a couple must always be applied to the beam with a finite distance between the forces which make it up. Actually the distance may be so small with respect to the length of the span to which the couple is applied that its effect is negligible, and so the couple is assumed applied at a point. In such a case the designer is tacitly assuming that the force W is infinite and that the value of b is zero, but that the product of infinity and zero is a positive constant, the couple. When it is desired to use Eqs. 36 in this manner, b becomes zero and the last term on the right side drops out. The equations are otherwise unchanged.

Care should be exercised, in using the equations, to remember that they give the restraining moments at the supports. When the couple moves up to the restrained support it is necessary to distinguish carefully between the restraining moment at the support and the bending moment out in the beam at the couple, a differential distance from the support. They are not the same.

Eqs. 36 may now be differentiated with respect to k, equated to zero, and criterions so developed for placing the couple in proper position for maximum bending moment at either support; thus, for maximum M_L—

$$k = \frac{1}{1 + C_R} \ldots (37)$$

and, for maximum M_R—

$$k = \frac{C_L}{1 + C_L} \ldots (38)$$

Eqs. 37 and 38 give the proper position of the couple in order to produce a truly mathematical maximum bending moment at the support. Eqs. 36, of course, are formulas of influence lines for the bending moments at the supports.

The mathematical maximums to which Eqs. 37 and 38 refer are at the points of zero slope of these curves. It is important to remember that these mathematical maximums are not always greatest bending moments. Greater bending moments than the mathematical maximums sometimes occur when the couple is placed at the supports. This happens in the case of a beam with fixed ends.

The positions of a couple, in order to produce zero bending moment at the support, may also be found readily by equating Eqs. 36 to zero, and solving for k. It is of interest to note that a couple occupies the same position in the span in order to produce zero bending moment at a support as would be occupied by a single concentrated load in order to produce maximum bending moment at the same support. Such positions accordingly may be read directly from Fig. 6.

It is also possible to develop equations for the bending moment out in the span, at differential distances from the forces that make up the couple. From these follow other interesting conclusions similar to the foregoing. Space limitations do not permit their inclusion herein.

10. Frame Characteristics and Their Computation

Equations have been developed for computing the bending moments due to loads placed on beams with restrained ends. Criteria have also been derived which place the loads properly to produce various kinds of maximum bending moments. The use of all of these requires a knowledge of the value of the transmission coefficients of the loaded span. If the effects of the loads are to be distributed to any frame, of which the loaded beam is a part, the designer must also know the transmission coefficients of the connecting beams and the moment splits at the joints. Often sidesway and lateral forces are involved which make it imperative to know the value of torque splits at the joints. All these terms—transmission coefficient, moment split, restraint index, and torque split—are included under the covering term, "frame characteristics."

Expressions have been developed for the value of the transmission coefficient and the moment split. Expeditious methods for computing the remaining characteristics, in order that the foregoing equations may become useful, are developed as follows.

The torque split is related very closely to the moment split, the only difference being the number of members that affect its value. The torque-split value for a member is that fraction by which any torque applied to a joint of a frame (artificially, as by a crank) must be multiplied in order to obtain the induced bending moment in that particular resisting member. The moment split does the same thing. The difference between the two lies in the fact that, in the case of the moment split, one of the radiating members of the joint is used as a vehicle for applying the torque, which is its own bending moment.

The member that applies torque, since it actuates the joint, cannot also participate as a resisting member. In the case of torque applied artifically to the joint, all the members act as resisting members. Thus, in the case of the moment split, all the members except one act as resisting or receiving members, and in the case of torque split all the members act as receiving members.

It is desirable, from the standpoint of ease of computing frame characteristics, that the expressions for torque split and for moment split be interrelated.

Writing R_B for the restraint index in bending for member 1 of a four-member joint, and R_T for the restraint index in torsion for the joint, from analogy with Eq. 9 for moment split, t_1 for torque split is:

$$t_1 = \frac{R_T}{i_1} = \frac{1}{i_1\left(\frac{1}{i_1}+\frac{1}{i_2}+\frac{1}{i_3}+\frac{1}{i_4}\right)} \quad \ldots\ldots\ldots\ldots\ldots (39)$$

However, when member 1 is the transmitting member, its restraint index is:

$$R_B = \frac{1}{\frac{1}{i_2}+\frac{1}{i_3}+\frac{1}{i_4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Therefore:

$$t_1 = \frac{1}{1+\frac{i_1}{R_{1B}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

Thus, when the restraint index has been computed for a member, the torque-split value for the same member may be found simply by dividing the stiffness index by the restraint index, adding 1 mentally, and taking the reciprocal of the result.

It is possible, of course, to compute frame characteristics for a small frame by using Eq. 5 directly for coefficients, Eq. 9 for moment splits, and Eq. 41 for torque splits. Such a procedure lacks order, and is conducive to errors. It has been found more satisfactory to use a tabular form of computation. In this form the equations are used, but they are used by rule, and more or less unconsciously.

Table 1 with its accompanying sketch will be used to illustrate this method of computation. It is designed with a view to keeping the computations in order, having them in such form that errors are reduced to a minimum, and rendering the procedure routine, as follows:

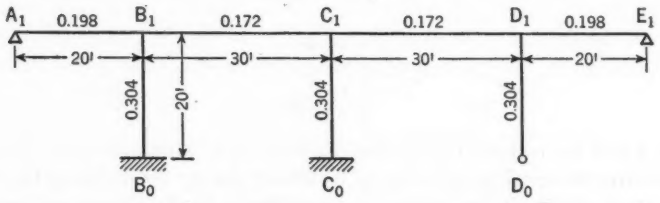
1. Enter all members of the successive joints in Col. 1, except joints at which the frame terminates at a fixed support, such as point B_0, or at a free support, such as point A_1. It is not necessary but is desirable to enter the members in the same order for all joints. Usually the member radiating downward is entered first, followed by successive members, clockwise around the joint in order. The letters that represent each member should read away from the joint, thus: C10 denotes the column $C_1\,C_0$; 1CB denotes the member $C_1\,B_1$; and 1CD denotes the member $C_1\,D_1$—all originating at joint C_1. Since in general each member connects with two joints, each will appear twice in the table, once under each of its two connecting joints.

2. Enter the Q-values for each member in Col. 2. They appear at the centers of members in the diagram of Table 1. These Q-values, as previously defined, are the ratios of the span length divided by the moment of inertia. For some purposes (such as the determination of angular rotations at the joint, these ratios should be computed with length and moment of inertia of the section in the same units. For the purpose of computing frame characteristics,

however, they need not be in the same units. If all lengths are in feet and moments of inertia are in inches, the computed characteristics will be the same as if all were in the same units. The essential requirement is that all members and joints be treated on the same basis.

3. At the right side of each Q-value entered in Col. 2, Table 1, place the appropriate quantity $(2 - C)$ in so far as the C-values are known. The transmission coefficient to be used for a particular member is that value of C which transmits moment away from the joint in connection with which the member

TABLE 1.—COMPUTATION OF FRAME CHARACTERISTICS



Member	Stiffness Index $Q(2-C) = i$		Restraint index R	Transmission Coefficient C (Eq. 5)		Moment Split m (Eq. 9)			Torque split, t (Eq. 41)
				Value	Member				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Joint B_1									
B10	0.304 (2 − 0.5)	0.456	0.1665	0.392	C_{B01}	(01)	0.387	0.465	0.268
1BA	0.198 (2 − 0)	0.396	0.1763	0.346	C_{1AB}	0.420	(AB)	0.535	0.308
1BC	0.172 (2 − 0.329)	0.287	0.212	0.310	C_{1CB}	0.580	0.613	(CB)	0.424
Joint C_1									
C10	0.304 (2 − 0.5)	0.456	0.146	0.403	C_{C01}	(01)	0.391	0.389	0.243
1CB	0.172 (2 − 0.310)	0.291	0.1785	0.329	C_{1BC}	0.502	(BC)	0.611	0.381
1CD	0.172 (2 − 0.294)	0.293	0.1775	0.330	C_{1DC}	0.498	0.609	(DC)	0.376
Joint D_1									
D10	0.304 (2 − 0)	0.608	0.1669	0.392	C_{D01}	(01)	0.395	0.321	0.215
1DC	0.172 (2 − 0.330)	0.288	0.240	0.294	C_{1CD}	0.579	(CD)	0.679	0.455
1DE	0.198 (2 − 0)	0.396	0.1955	0.334	C_{1ED}	0.421	0.605	(ED)	0.330

appears. In the case of the frame in Table 1 the coefficients of connecting members toward joints A_1, B_0, C_0, D_0, and E_1 are all known since these joints are either free or fixed. These known coefficients will be sufficient to solve the entire frame in this case. When sufficient coefficients are not so known, it is necessary to estimate some of them, as will be demonstrated subsequently.

4. Compute the i-values for all members whose C-values away from the joint are known. In Table 1, this may be done at once for members B10 and 1BA for joint B_1; member C10 for joint C_1; and members D10 and 1DE for joint D_1 (Col. 3, Table 1).

5. At some of the joints all i-values except one should be known. For the frame in Table 1, this is true of joints B_1 and D_1. The R-value, for that member, whose i-value is missing at each joint, may now be computed, since it

depends upon all the i-values except its own. This is evident from an examination of Eq. 4 in Section 2. Eq. 40 may be written into an equivalent form:

$$R = \frac{i_2}{1 + \frac{i_2}{i_3}\left(1 + \frac{i_3}{i_4}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

The order in which the i-values appear in Eq. 42 is immaterial. It is convenient to use the lowest value in the numerator and the successively higher values in order. For three-member joints, such as the present example, the parentheses in the denominator of Eq. 42 drop out. Using joint B_1 as an example, the i-values for members B10 and 1BA are known. Under the number 0.396 on the slide rule the divisor 0.456 is set. To the quotient 0.869, unity is added mentally, and the divisor 1.869 is set under 0.396 to give the proper restraint index, 0.212 as entered in Col. 4, Table 1, for member 1BC.

6. For all members, except those at a two-member joint, the R-value is checked at once by computing the moment-split values for its member. In this case the R-value 0.212 is divided by the i-value of member B10 to give the moment-split value for this member as 0.465. It is next divided by 0.396 to yield the moment split for member 1BA as 0.535. These moment splits are for member 1CB, when it transmits moment toward the joint, as indicated in Col. 9. It is seen that the sum of 0.465 and 0.535 is unity. This would not have been true if there had been an error in computing the R-value from the two i-values.

7. The C-value for member 1CB may now be computed with confidence, since the R-value upon which it depends has been checked. The R-value computed for member 1BC was derived from i-values which involve coefficients away from joint B_1. The derivation of Eq. 5 will show that the transmission coefficient that results from this value of R is then toward the joint. The subscripts for C are accordingly reversed in entering them in Col. 6. The coefficient value 0.310 is computed by dividing the R-value 0.212 by the Q-value of the member, 0.172, adding 2 mentally, and taking the reciprocal of the result.

8. The value of C_{1CB} computed in step 7 is now substituted in its proper place under joint C_1. From this the i-value for the member 1CB may be computed. All the i-values at this joint are now known, except for member 1CD. Its R-value of 0.1775 may then be computed, and from it the moment splits, for the member, and the value of C_{1DC} just as was done in step 7. The new coefficient is substituted under joint D_1 and the operations repeated until all coefficients and moment splits ore obtained.

9. Torque splits may now be computed by Eq. 41, if the effects of sidesway and wind moments are to be analyzed. Otherwise, the torque-split values will not be needed. The value of 0.268 in Col. 10, Table 1, is found by taking the ratio (on the slide rule) of 0.456 to 0.1665, adding unity mentally, and taking the reciprocal of the result. The torque splits for the joint must add to unity just as the moment splits, which is a check on computations. Other torque splits are found in exactly the same manner, always dealing with the i-values and R-values in any single horizontal line.

11. The Estimation of the Values of Coefficients

It frequently occurs that not enough coefficients are known in advance of computing the frame characteristics to permit the analyst to begin a tabular computation. Also it may be necessary occasionally to investigate the adequacy of a particular beam of a frame, without computing the characteristics of the entire frame. In such cases it is highly desirable to be able to make an accurate estimate of the value of the coefficients for a particular beam. This can be done readily with an accuracy which is ordinarily within 1%; and in any case the error is never great enough to affect the value of the coefficient beyond the third place to the right of the decimal point.

Eq. 5, for the value of the coefficient, may be expanded into the following form for a four-member joint:

$$C_1 = \frac{1}{2 + \frac{1}{Q_1} \frac{1}{\frac{1}{Q_2\,(2 - C_2)} + \frac{1}{Q_3\,(2 - C_3)} + \frac{1}{Q_4\,(2 - C_4)}}} \quad \ldots\ldots (43a)$$

The expressions for the i-values of the restraining members all contain the quantity $2 - C$. It is true that these C-values are all different. It is readily conceivable, however, that there is some kind of an equivalent C which, if inserted uniformly into the three parentheses in the denominator of Eq. 43a, would give the same result for the coefficient being computed by the equation as if the exact C-value for each member had been used. With this concept in mind, this equivalent coefficient may be called C_e, and Eq. 43a can be written:

$$C_1 = \frac{1}{2 + \frac{1}{Q_1} \frac{2 - C_e}{\frac{1}{Q_2} + \frac{1}{Q_3} + \frac{1}{Q_4}}} = \frac{1}{2 + \frac{1}{Q_1} \frac{Q_2\,(2 - C_e)}{1 + \frac{Q_2}{Q_3}\left(1 + \frac{Q_3}{Q_4}\right)}} \quad \ldots\ldots (43b)$$

Either part of Eq. 43b may be used for computation. The right-hand part has been arranged for convenient use with the slide rule, as was done for computing restraint indexes in Eq. 42, step 5, Section 10.

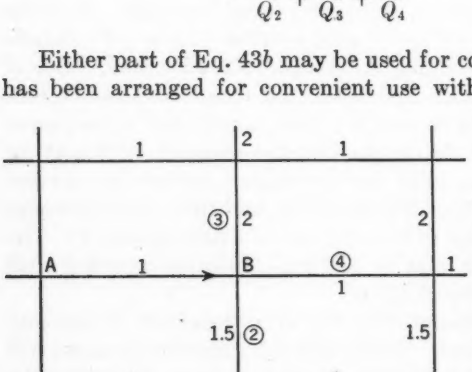


Fig. 9

Let it be required to estimate the value of the coefficient toward the right, of member AB, of the beam shown in Fig. 9. The Q-values of the beam itself, and of the restraining beams on the right, are shown by the numbers, 1, 1.5, 2, etc., in the centers of the spans. The restraining members have been given the numbers shown in circles.

For estimating coefficients in restraining members away from joint B, certain principles should be kept in mind:

1. A stiff member connecting into a limber member will have a low coefficient;
2. Many members, acting as a restraint, tend to raise the coefficient;
3. A member, restrained by a single member of its own stiffness, has a coefficient of approximately 0.29; and
4. The maximum value of the coefficient is 0.5 and the minimum value is 0.0.

By inspection note that restraining member 2 connects into members, at its end away from joint B, two of which are stiffer than, and one as stiff as, itself. One may judge its coefficient away from joint B to be approximately 0.4. Likewise, the coefficient for member 3 is perhaps about the same, and for member 1 about 0.3. Estimating that C_e is 0.35 and inserting it in Eq. 43*b*, in order to obtain a close approximation of C_{AB}:

$$C_{AB} = \frac{1}{2 + \frac{1}{1}\dfrac{1\,(2 - 0.35)}{1 + \frac{1}{1.5}\left(1 + \frac{1.5}{2}\right)}} = \frac{1}{2 + 1.65 \times 0.461} = 0.363.$$

This estimate will be accurate within the third decimal place. It can be seen readily that a considerable error may be made in C_e without great effect on C_{AB}. Thus, if the analyst had assumed C_e to be 0.4 instead of 0.35, the value of C_{AB} would be 0.366, and, if he had assumed 0.3 instead of 0.35, C_{AB} would be 0.359.

Such estimated coefficients may be used with entire confidence in computing frame characteristics. The exact value of the coefficient will be obtained, in the course of the computation, and finally may be used instead of the estimated value. No further correction need be made for errors in estimation, however, unless the exact value should differ from the estimated value beyond the third decimal place. In the writer's experience this does not occur.

These estimated coefficients are sufficiently accurate to be used in connection with the equations for restraining moment, in order to make spot checks of particular beams under special loads.

12. The Sense of the Bending Moment

It is obvious from the frame characteristics of Table 1 that bending moment may be distributed to the frame at a single operation. Before this is done, it is necessary to define the sense of the bending moment in such a manner as to avoid confusion. The concept of positive and negative bending moment has been abandoned for the purposes of making moment distribution, for the following reason: In courses in mechanics of materials, bending moment is defined as positive for horizontal beams when the beam is concave upward and negative when the beam is concave downward. These bending moments are also defined as positive when the algebraic sum of moments of forces on the left of a section is clockwise about the section and negative if counterclockwise, the definitions being reversed when forces on the right are considered.

Such definitions serve well for horizontal beams, but when the beams radiate in all directions, as they do in a frame, these definitions become confusing. Even though a proper distinction of sense of bending moment is developed for the frame in terms of plus signs and minus signs, such a distinction is likely to be confused with the old concept of plus and minus bending moment, which was so well learned in elementary studies.

For these reasons bending moment is designated as "normal" or "reverse." "Normal bending moment" is defined as follows (see Fig. 10(*b*)):

I. Bending moment is said to be normal at the end of a beam if, when the beam is cut from its frame and considered as a free body, a clockwise torque must be applied at the end to hold the beam in the same condition of stress to which it was subjected in the frame.

"Reverse bending moment" is defined as follows (see Fig. 10(*a*)):

II. Bending moment is said to be "reverse" at the end of a beam if, when the beam is cut from its frame and considered as a free body, a counterclockwise torque must be applied to the end in order to hold the beam in the same condition of stress to which it was subjected in the frame.

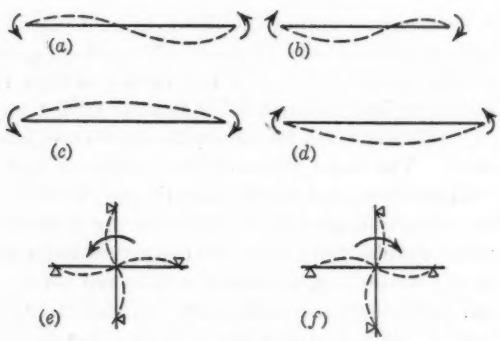


FIG. 10

Both definitions are consistent without regard to the end of the beam to which the torque is applied. Figs. 10(*c*) and Fig. 10(*d*) show bending moments of opposite senses at the two ends of the two members. These senses of the bending moment may be defined equally well in terms of the joint from which the member radiates. Thus, for normal bending moment (see Fig. 10(*e*)):

III. Bending moment is said to be normal in the end of a member when the member is so flexed at that end as to resist the effect of a clockwise torque, artificially applied to its connecting joint.

For reverse bending moment (see Fig. 10(*f*)):

IV. Bending moment is said to be reverse at the end of a member when the member is so flexed at that end as to resist the effect of a counterclockwise torque artificially applied to the joint to which the member end connects.

13. Distribution of Bending Moment to the Frame

Bending moment may be distributed to the frame at a single operation by the following rules:

1. Determine the sense of the bending moment at the ends of the beam of origin according to definitions I and II or III and IV.

2. Multiply the bending moment at the end of the beam of origin by the moment-split values for each of the receiving members, the member of origin being considered the transmitting member, and so obtain the part of the bending moment induced in each. Change the sense of the bending moment as it passes through the joint at the end of the member of origin and enter each portion of the moment for the proper receiving member.

3. Each of the bending moments, as obtained in rule 2, is now multiplied by that transmission coefficient, of the member in which it occurs, which transmits moment away from the joint, in order to obtain the induced bending moment at the opposite end of the member. It is entered here without change in sense.

4. At each succeeding joint proceed as in rule 2 and repeat rule 3 after the moment passes through the new joint.

5. The distribution of the moment is completed under the following conditions:

(*a*) When it reduces to a negligible portion of its value at the origin;
(*b*) When it arrives at a fixed or free support; and
(*c*) When, as in most cases, it is carried only two joints beyond the joint of origin.

6. In order that all joints may remain balanced, distribution should cease just after the moment has passed through the last joint, not as it arrives.

An example will serve to illustrate the application of rules 1 to 6 for the distribution of bending moment, as well as to sum up the methods of computing bending moments in restrained beams. Let it be required to place two moving loads of the highway H-10 type on the two spans 1BC and 1CD in Table 1 each in position for maximum bending moment at joint C_1. Let it then be required to compute the two maximum bending moments and the simultaneous bending moments at the opposite ends of the respective beams, and to distribute these four bending moments to the frame.

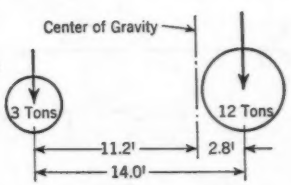


Fig. 11

The loading type is shown in Fig. 11, with the center of gravity located. The computation follows: $U = 3 \times 11.2^2 + 12 \times 2.8^2 = 376.3 + 94.1 = 470.4$; $S = 376.3 \times (-11.2) + 94.1\ (2.8) = -4{,}214 + 263 = -3{,}951$; λ^2 (either span) $= \dfrac{470.4}{15 \times 900} = 0.0348$; $\lambda = 0.187$; and $\epsilon^3 = \pm \dfrac{3{,}951}{15 \times 27{,}000} = \pm 0.00975$ (sign depends on orientation).

From Fig. 6, using the coefficients from Table 1 (see also Fig. 12), the maximum bending at joint C_1 is: In span 1BC, $k_R = 0.580$; and in span 1DC,

$k_L = 0.422$. Entering Fig. 5, with these values and the value of λ:

Symbol	Span 1BC	Span 1DC
B	0.323	0.302
D	0.302	0.323
A_L	$\frac{0.310}{1 - 0.310 \times 0.329} = 0.345$	$\frac{0.330}{1 - 0.330 \times 0.294} = 0.365$
A_R	$\frac{0.329}{0.898} = 0.366$	$\frac{0.294}{0.903} = 0.326$

The two loads have been shown, properly located with respect to the two joints B_1 and C_1, in the respective spans 1BC and 1CD in Fig. 12.

Next, with Eqs. 27, compute the bending moments induced at the right and left of the loaded spans: In span 1BC—

$$M_{1BC} = -15 \times 30 \times 0.345\,[0.302 - 0.0098 - 0.329\,(0.323 + 0.0098)] = -28.4 \text{ ft-tons}$$

$$M_{1CB} = -15 \times 30 \times 0.366\,[0.323 + 0.0098 - 0.310\,(0.302 - 0.0098)] = -39.9 \text{ ft-tons (due to placing for maximum moment)}$$

and in span 1CD—

$$M_{1CD} = -15 \times 30 \times 0.365\,[0.323 + 0.0098 - 0.294\,(0.302 - 0.0098)] = -40.5 \text{ ft-tons (due to placing for maximum moment)}$$

$$M_{1DC} = -15 \times 30 \times 0.326\,[0.302 - 0.0098 - 0.330\,(0.323 + 0.0098)] = -26.8 \text{ ft-tons.}$$

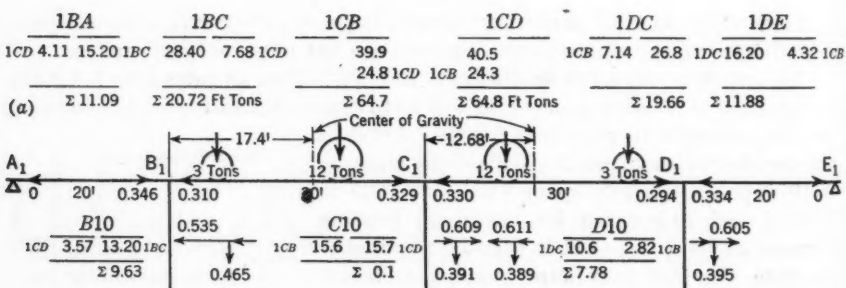


FIG. 12

These bending moments may now be entered in Fig. 12, for distribution to the frame. For this purpose it is conventional to place normal bending moment on the right and reverse bending moment on the left. Two columns will be seen in Fig. 12 for each member end. For horizontal beams, under gravity loading, the bending moment is always reverse on the left of the loaded span

and normal on the right. The bending moments computed will be seen in Fig. 12 according to this classification.

The moment-split values which are required for the purpose have been copied from Table 1, and are shown adjacent to the joints of the loaded beams, with arrows indicating the direction of moment flow.

As an example, consider the moment 28.4 ft-tons, at the left end of span 1BC. First multiply it by the moment-split value 0.465 to obtain 13.20 ft-tons as the moment at B10. This is entered as normal moment since it has passed through a joint. The same moment multiplied by 0.535 gives the value 15.20 in beam 1BA which is also entered as normal moment. The 13.20 is now multiplied by the coefficient 0.5 of its member away from B_1, to give 6.60 also normal at the support end B_0.

In a similar manner, 39.9 is multiplied by 0.609 and by 0.391 to give the reverse moments it induces in members 1CD and C10. These are entered, as are the values obtained from the 40.5 ft-ton moment times its appropriate splits, and those from the 26.8 ft-ton moment. It should be particularly noted that, in the beginning of its distribution, moment flows only outward from the loaded span of origin, and does not cross that span.

In placing the distributed moments, it will be seen that the point of origin has been noted beside each. This is only precautionary to provide for rechecking in case of possible necessity.

When the moments are all distributed, the algebraic sum is taken, at each member end, and noted by the summation sign below as the final bending moment at that point. For instance, the 20.72 ft-tons of reverse moment at member 1BC is equal to 28.4 − 7.68. The algebraic sum of all the total moments at any joint must add to zero. At joint B_1 the sum of the two normal moments 9.63 and 11.09 is equal to the reverse moment in member 1BC of 20.72. This is a check on the net bending moments as indicated by the summation signs.

The method outlined is entirely satisfactory for small frames. When large frames, involving many members and joints, are to be solved, a much more comprehensive method is used. This method involves a moment-distribution chart for each joint. Its bookkeeping arrangement is so designed that there is a minimum chance of error. One distribution also is sufficient for all purposes, including live-load and dead-load moment distribution, wind moments and sidesway, as well as loading for maximum moment at every point in the frame. It is too lengthy to include in the present paper.

14. Coordination with the Method of Successive Approximations

The method of solving rigid frames by successive approximations, as proposed by Hardy Cross,[2] M. Am. Soc. C. E., starts, as does the slope deflection method, with fixed-end moments for loaded members. As a result there can be little coordination of the methods of computing moments in loaded members. The Cross method of distributing moments to a frame can be shown to be a

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

special case of the method here outlined. That method consists essentially of the distribution, at a single joint, of an opposing torque equal to the unbalance at that particular joint; the outer ends of connecting members being considered as fixed, or in case they connect into a pinned support, as freely supported. This partial distribution is made successively, a single joint at a time, until error diminishes to a negligible amount.

The Cross method then involves a torque split for a joint, whose members connect into either fixed or free supports at their outer ends, and transmission coefficients toward fixed ends or free ends, which were previously pointed out to be either 0.5 or 0. It will be remembered that, in the Cross method, the value of I/L is termed K, which is the reciprocal of the Q-value used in this paper.

The expression for the torque split, Eq. 39, Section 10, may be expanded as follows, for a four-member joint—

$$t_1 = \frac{1}{Q_1 (2 - C_1)} \times \frac{1}{\frac{1}{Q_1 (2 - C_1)} + \frac{1}{Q_2 (2 - C_2)} + \frac{1}{Q_3 (2 - C_3)} + \frac{1}{Q_4 (2 - C_4)}} \quad \ldots (44a)$$

Assume that members 1, 2, and 3 connect to fixed or locked joints at their outer ends and that member 4 connects to a pinned or free support, the transmission coefficients or carry-over factors for the first three members will be 0.5 and for the fourth 0.0. Inserting these values in Eq. 44*a* and appropriate values of K for the reciprocals of $Q \left(= \frac{1}{L/I} = K \right)$: $t_1 = \frac{K_1}{1.5} \times \frac{1}{\frac{K_1}{1.5} + \frac{K_2}{1.5} + \frac{K_3}{1.5} + \frac{K_4}{2.0}}$; or:

$$t_1 = \frac{K_1}{K_1 + K_2 + K_3 + 0.75\, K_4} \quad \ldots (44b)$$

Eq. 44*b* will be recognized as the well-known Cross expression for what has been called torque-split value in this paper. The equation is written to apply to the particular joint under discussion, with three members fixed at the outer ends, and the fourth freely supported.

15. Other Applications of the Transmission Coefficient

The uses of the concept of the transmission coefficient, which have been outlined in this paper, represent only a very limited number of the purposes to which it has been adapted. It may be of interest to list some of its other applications.[3,4] It has been adapted to the placing of moving loads for greatest

[3] "The Solution of Statically Indeterminate Structures," by R. C. Brumfield, 1933 (printed in mimeograph form by Cooper Union, New York, N. Y.).

[4] "Restrained Beams and Rigid Frames," by R. C. Brumfield, 1941 (revision and expansion of "The Solution of Statically Indeterminate Structures"[3]; printed in mimeograph form by Cooper Union, New York, N. Y.).

shear both at the support and in the span, just as was done for maximum bending moment. An equation for elastic deformation has been developed from its use, which is perfectly general and applies to the entire beam, irrespective of the intervention of concentrated loads and supports. This equation has certain important uses. The transmission coefficient has been developed for straight beams of variable section, with restrained ends. For such beams its values theoretically may become infinite, but actually do not ordinarily exceed unity. It has been applied to curved beams, in which the coefficient is often negative. Curved beams, in fact, are reduced to the same status as straight beams, the formulas for the former reducing to those of the latter when curvature disappears. Moment splits, torque splits, and the effects of a movement of the supports may be computed for curved beams. The concept enables the beam which is a member of a noncoplanar frame to be solved by the formulas outlined herein. The number of factors that affect coefficients of such a beam is greater than that for coplanar beams; but, once the coefficients are computed, a beam in a noncoplanar frame may be loaded in quite the same manner as outlined for a coplanar straight beam, and moments computed by the same formulas.

16. Acknowledgment

The methods outlined, as well as most of those indicated, have been thoroughly tested in professional engineering practice, in and around the City of New York, N. Y. In this process a great many valuable suggestions have been received, for which acknowledgment is made.

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PAPERS

MATHEMATICAL EXAMINATION OF THE HIGHWAY TRANSITION SPIRAL

BY JOHN ORAN EICHLER,[1] ASSOC. M. AM. SOC. C. E., AND HOWARD W. EVES,[2] ESQ.

SYNOPSIS

The growing importance of superhighways demands that increased attention be given to the inherent problems of construction and design. One of the most important of these problems is that of horizontal curvature and the attendant superelevation. A rational solution of this problem is the adaptation of a transition spiral, whose curvature varies inversely as the arc length. From considerations of speed, safety, and comfort it is found that two cases arise—that in which a central piece of circular arc is required, and that in which the curve is transitional throughout. The second case is desirable on grounds of relative simplicity of calculations, formulas, and field work. The question then naturally arises: "When, under certain given limiting conditions, can a curve be transitional throughout?" An answer to this question (and to other office and field problems related to the spiral) necessitates an examination of the mathematics of the transition spiral. Most existing mathematical treatments seem to suffer shortcomings due to lack of proof, or to failure to consider comfort and safety, or to loss of generality, or to confusion between exact and approximate relations, or to failure to be concise. Therefore, it is felt that an exposition of the curve and its properties obviating these faults and conducted primarily as a basis for examining questions, such as the foregoing, is eminently desirable. Such an attempt is offered in this paper.

1. INTRODUCTION

It seems safe to predict that the road engineers of the postwar period will have to give ever-increasing attention to the problems connected with the

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by October 1, 1945.

[1] Asst. Prof., Civ. Eng., College of Applied Science, Syracuse Univ., Syracuse, N. Y.

[2] Asst. Prof., Applied Mathematics, Syracuse Univ., Syracuse, N. Y.

development of superhighways. The aim will be to construct a roadway capable of carrying high-speed traffic, designed for maximum safety and comfort to drivers. Of the many problems involved, that of horizontal curvature, accompanied with proper superelevation, is one of the most important, and it is essentially with this feature of the future road that this paper is concerned. In particular, the writers would like to offer a simple and definite answer to the question: "When can a curve be transitional throughout?" Considerable theoretical research, table construction, and discussion have already taken place, as a complete bibliography of the subject would testify; but, because of the scattered nature of these references, the complete theory of easement curves is not readily available to the average road engineer. It would seem desirable, therefore, to have a clear, concise, and easily accessible account of the mathematical theory of the ideal easement curve, since it is only through a sound grasp of the mathematical properties of this curve that the dependent practical field work of laying out easement curves can be conducted intelligently. A representative bibliography is included as an Appendix to the paper.

2. The Transition Spiral

The forces acting on a vehicle and its passengers are altered when the vehicle passes from a straight to a curved path. The value of the centrifugal force P acting on a body of weight W, traveling at a speed V, and rounding a curve of radius r, is

$$P = \frac{W V^2}{g r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which g is the acceleration due to gravity. If the path of the vehicle changes immediately from a straight line to a circle of radius r, the force P will be applied instantaneously, and the vehicle and passengers will be subjected to a proportionate shock. However, if the radius of curvature of the path is gradually reduced from infinity to its minimum value, the centrifugal force P will be increased gradually and little shock will be experienced by the vehicle and its passengers. Any curve that provides this transition is called an "easement curve." J. Glover (9)[3] and W. H. Shortt (27) have argued that the rational way to achieve this transition is to make the rate of gain of the "centrifugal ratio," P/W, constant throughout the curve. This is equivalent to the demand that the driver sharpen his curve by turning his steering wheel at a constant rate. Since, for a fixed speed V, P/W is proportional to $1/r$, it follows that the mathematical requirement for the perfect easement curve is that the radius of curvature, r, at any point should be inversely proportional to the distance, s, of that point from the beginning of the curve. Such an easement curve may be termed a "transition spiral." For a fixed speed, superelevation slope is proportional to curvature and hence, for the transition spiral, to s. It follows, therefore, that this curve is also ideal for the application of superelevation.

[3] Numerals in parentheses, thus: (9), refer to corresponding items in the Bibliography (see Appendix).

Many other forms of easement curves have been suggested and used, such as empirical or bastard transitions (possessing little to recommend them), the quadratic parabola (advocated by E. L. Leeming (16)), the cubic parabola (used extensively for short transitions in railroad work), the lemniscate (developed and used by F. G. Royal-Dawson (22) in England), the multi-compound spirals (such as the one evolved by W. H. Searles (25)), and the ten-chord spiral (13) adopted by the American Railway Engineering Association (which is essentially a transition spiral laid out in ten equal chord lengths). All these curves are modifications or approximations of the theoretically ideal transition spiral. It is not proposed here to enter into the arguments favoring any one easement curve over the others, but to develop the mathematics of the transition spiral and then, in the interests of economy in calculation and field work, to examine briefly some approximate formulas derived from the exact ones.

3. Intrinsic Equation of Spiral

In Fig. 1, let O be the beginning of the spiral on the initial tangent or x-axis. Then, if s is the arc length of the spiral from O to any point P on the curve, r is the radius of curvature at point P and ϕ is the angle that the tangent at point P makes with the x-axis, by the definition of a transition spiral:

$$\frac{1}{r} = k\,s \qquad (2)$$

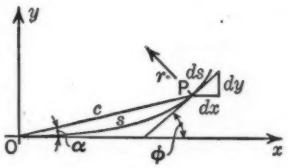


FIG. 1

in which k is a constant of proportionality; but, since curvature is the arc rate of change of ϕ: $\frac{1}{r} = \frac{d\phi}{ds} = k\,s$; whence, at the end of the spiral,

$$s = K\sqrt{\phi} \qquad (3)$$

in which

$$K = \sqrt{\frac{2}{k}} = \sqrt{2\,R\,S} \qquad (4)$$

R and S being the final radius of curvature and length of the spiral, respectively.

4. Parametric Cartesian Representation

Letting ds, dx, and dy be differential changes in s, x, and y, respectively:

$$dx = ds\cos\phi = 0.5\,K\,\phi^{-1/2}\cos\phi\,d\phi \qquad (5a)$$

and

$$dy = ds\sin\phi = 0.5\,K\,\phi^{-1/2}\sin\phi\,d\phi \qquad (5b)$$

whence

$$x = 0.5\,K\int_0^{\phi}\phi^{-1/2}\cos\phi\,d\phi$$

$$= K\,\phi^{1/2}\left[1 - \frac{\phi^2}{5(2!)} + \frac{\phi^4}{9(4!)} - \frac{\phi^6}{13(6!)} + \cdots\right] \qquad (6a)$$

and

$$y = 0.5\,K \int_0^{\phi} \phi^{-1/2} \sin \phi \, d\phi$$

$$= K\,\phi^{1/2}\left[\frac{\phi}{3} - \frac{\phi^3}{7(3!)} + \frac{\phi^5}{11(5!)} - \frac{\phi^7}{15(7!)} + \cdots\right] \ldots\ldots\ldots (6b)$$

5. Polar Deflection Angle α

Let α be the polar deflection angle of point P with respect to O as origin and the tangent to the curve at O as x-axis. Then, by Eqs. 6,

$$\tan \alpha = \frac{y}{x} = \frac{\frac{\phi}{3} - \frac{\phi^3}{7(3!)} + \frac{\phi^5}{11(5!)} - \frac{\phi^7}{15(7!)} + \cdots}{1 - \frac{\phi^2}{5(2!)} + \frac{\phi^4}{9(4!)} - \frac{\phi^6}{13(6!)} + \cdots}$$

$$= \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5{,}997} - \frac{\phi^7}{198{,}728} - \cdots \ldots\ldots\ldots\ldots\ldots (7)$$

Setting

$$C = \frac{\phi}{3} - \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

one may now compute the following values, expressing C, in minutes, corresponding to given values of ϕ, in degrees:

ϕ, in degrees	C, in minute
15	0.2
20	0.4
25	0.8
30	1.4
35	2.2
40	3.4
45	4.8
50	6.6

Such a table is given by J. Barnett, M. Am. Soc. C. E. (2), and is valuable for quickly obtaining the corrections C which must be added to one third of any given ϕ in order to obtain the corresponding α. Use of this table will be made in section 9, where is developed the program for finding the control data of a required spiral.

6. Chord Length c

Denoting the chord length corresponding to arc s by c:

$$c = (x^2 + y^2)^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Substituting from Eqs. 6:

$$c = K\,\phi^{1/2}\left[\,1 + \frac{\phi^4}{25(2!)^2} + \frac{\phi^8}{81(4!)^2} - \frac{2\,\phi^2}{5(2!)} + \frac{2\,\phi^4}{9(4!)} - \frac{2\,\phi^6}{13(6!)} + \frac{2\,\phi^8}{17(8!)} - \frac{2\,\phi^6}{45(2!)(4!)} + \frac{2\,\phi^8}{65(2!)(6!)} + \cdots + \frac{\phi^2}{9} + \frac{\phi^6}{49(3!)^2} - \frac{\phi^4}{21(3!)} + \frac{2\,\phi^6}{33(5!)} - \frac{2\,\phi^8}{45(7!)} - \frac{2\,\phi^8}{77(3!)(5!)} + \cdots\,\right]^{1/2} \quad \ldots\ldots (10a)$$

or

$$c = K\,\phi^{1/2}\left[\,1 + \phi^2\left(\frac{1}{9} - \frac{2}{5(2!)}\right) + \phi^4\left(\frac{1}{25(2!)^2} + \frac{2}{9(4!)} - \frac{2}{21(3!)}\right) + \phi^6\left(-\frac{2}{13(6!)} - \frac{2}{45(2!)(4!)} + \frac{1}{49(3!)^2} + \frac{2}{33(5!)}\right) + \cdots\,\right]^{1/2} \quad \ldots (10b)$$

Eq. 10*b* is in the form

$$c = s\,[1 + (A\,\phi^2 + B\,\phi^4 + C\,\phi^6 + \cdots)]^{1/2} \quad \ldots\ldots (11a)$$

or, expanding Eq. 11*a* by the binomial theorem,

$$c = s\left[\,1 + \frac{(\ )}{2} - \frac{(\ )^2}{8} + \frac{(\ )^3}{16} - \cdots\,\right] \quad \ldots\ldots (11b)$$

This then gives

$$c = s\left[\,1 + \phi^2\left(\frac{A}{2}\right) + \phi^4\left(\frac{B}{2} - \frac{A^2}{8}\right) + \phi^6\left(\frac{C}{2} - \frac{A\,B}{4} + \frac{A^3}{16}\right) + \cdots\,\right] \quad \ldots (12a)$$

or

$$c = s\,[1 - 0.0444444\,\phi^2 + 0.0007054\,\phi^4 - 0.0000025\,\phi^6 + \cdots] \quad \ldots (12b)$$

in which ϕ, of course, is given in radians. If ϕ is to be given in degrees:

$$c = s\,[1 - 1.354\,(10^{-5})\,\phi^2 + 6.6\,(10^{-11})\,\phi^4 - \cdots] \quad \ldots\ldots (13)$$

Setting

$$p = \frac{c}{s} \quad \ldots\ldots (14)$$

one may use Eq. 12*b* or Eq. 13 for constructing a useful table (or a graph, as has been done by H. J. Collins and C. A. Hart, Assoc. Members, Am. Soc. C. E. (5)) giving the values of p corresponding to values of ϕ.

7. Safety and Comfort Requirements for Driving the Spiral

(*A*) The centrifugal ratio $\frac{P}{W} = \frac{V^2}{g\,r}$ must have an upper limit ρ, for the following reason: Let μ be the angle of superelevation and ν the angle of friction for the tires on the road. Then, from mechanics,

$$\frac{V^2}{g\,r} = \tan(\mu + \nu) = \frac{\tan\mu + \tan\nu}{1 - \tan\mu\,\tan\nu} = \frac{e + f}{1 - e\,f} \quad \ldots\ldots (15)$$

in which $e = \tan \mu$ and $f = \tan \nu$. For practical reasons the superelevation slope e must be limited, and f (which is the coefficient of friction for transverse sliding of tires on the roadway) seems to depend on the speed. According to present American practice e seldom exceeds about 0.10, and, from a number of driving tests (19), it seems that f may be assumed at about 0.14 for a speed of 70 miles per hr to perhaps 0.08 for 100 miles per hr. According to Messrs. Collins and Hart (5):

> "The maximum centrifugal ratio for railways is normally taken as 1/8, but owing to the lower centre of gravity of motor vehicles, it may be safely increased to 1/4 for road work. This value is quite arbitrary and the 'factor of safety' will depend on a great number of considerations such as degree of superelevation, adhesion between road and tyres, bumpiness of road, etc."

(*B*) The rate of change of radial acceleration must have an upper limit γ. Let t be the time required to travel the entire length S of the spiral. Then

$$\gamma = \frac{V^2/R}{t} = \frac{V^2/R}{S/V} = \frac{V^3}{R\,S}\left(\frac{\text{ft}}{\text{sec}^3}\right).$$

This average rate varies for different drivers and many tests are needed to determine the maximum average rate. From considerations of comfort, Mr. Shortt (27) has fixed γ at $1\ \frac{\text{ft}}{\text{sec}^3}$, but more recent experiments by R. A. Moyer, Assoc. M. Am. Soc. C. E. (19), indicate that $\gamma = 2\ \frac{\text{ft}}{\text{sec}^3}$ is both safe and comfortable.

Paragraphs (*A*) and (*B*) yield the formulas,

$$R = \frac{V^2}{g\,\rho} \qquad (16a)$$

$$S = \frac{V^3}{R\,\gamma} = \frac{V\,g\,\rho}{\gamma} \qquad (16b)$$

and

$$\Phi = \frac{S^2}{K^2} = \frac{S}{2\,R} = \frac{V^3}{2\,R^2\,\gamma} = \frac{g^2\,\rho^2}{2\,\gamma\,V} \qquad (16c)$$

in which Φ is the deflection angle, R the radius of curvature, and S the arc length of the spiral for the point where the maximum centrifugal ratio is reached.

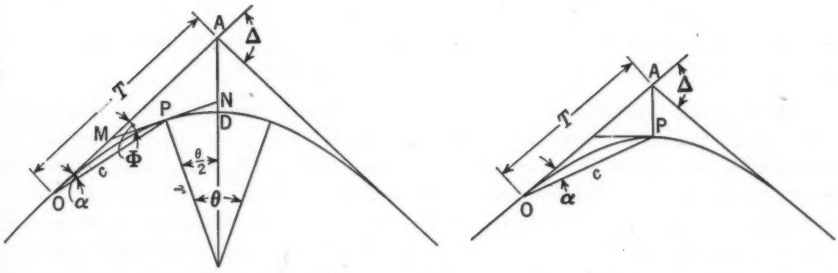
8. Computation of Tangent Distance T and External Distance E

Let Δ be the intersection angle of the highway curve. For a given velocity V and selected values of ρ and γ, Eq. 16*c* indicates that Φ cannot exceed $\frac{g^2\,\rho^2}{2\,\gamma\,V}$. Hence, in designing a curve for a given speed V and selected values of ρ and γ, two cases need to be considered—that for which $\Phi < 0.5\,\Delta$ and that for which $\Phi \geqq 0.5\,\Delta$. In the first case the curve must contain a central circular arc; and, in the second case, the curve will be transitional throughout.

Thus, as an example, for a velocity of 100 miles per hr, $g = 32 \frac{\text{ft}}{\text{sec}^2}$, $\rho = 0.25$, and $\gamma = 2 \frac{\text{ft}}{\text{sec}^3}$, Eq. 16*c* yields $\Phi = 0.4364$ radian, or about 25°. Hence, if $\Delta \leqq 50°$, the curve, under these conditions, will be transitional throughout; otherwise a central circular arc will be needed.

Case 1. $\Phi < 0.5\,\Delta$ (*Central Circular Arc Required*).—In Fig. 2(*a*), angle MAN = $90° - 0.5\,\Delta$ and angle MNA = $90° + 0.5\,\Delta - \Phi$. Also,

$$T = \overline{OM} + \overline{MA} = \frac{c \sin (\Phi - \alpha)}{\sin \Phi} + \frac{\overline{MN} \sin (90° + 0.5\,\Delta - \Phi)}{\sin (90° - 0.5\,\Delta)} \quad \ldots (17a)$$



(*a*) CASE 1, $\Phi < 0.5\,\Delta$ (*b*) CASE 2, $\Phi \geqq 0.5\,\Delta$

FIG. 2

Again,

$$\overline{MN} = \overline{MP} + \overline{PN} = \frac{c \sin \alpha}{\sin \Phi} + R \tan 0.5\,\theta \quad \ldots (17b)$$

Therefore,

$$T = \frac{c \sin (\Phi - \alpha)}{\sin \Phi} + \frac{\left(\frac{c \sin \alpha}{\sin \Phi} + R \tan 0.5\,\theta \right) \sin (90° + 0.5\,\Delta - \Phi)}{\sin (90° - 0.5\,\Delta)} \quad \ldots (18)$$

Furthermore,

$$\overline{AN} = \overline{MN} \frac{\sin \Phi}{\sin (90° - 0.5\,\Delta)} = \frac{c \sin \alpha + R \tan 0.5\,\theta \sin \Phi}{\sin (90° - 0.5\,\Delta)} \quad \ldots (19a)$$

and

$$\overline{ND} = R \sec 0.5\,\theta - R \quad \ldots (19b)$$

Combining Eqs. 19,

$$E = \overline{AN} + \overline{ND} = \frac{c \sin \alpha + R \tan 0.5\,\theta \sin \Phi}{\sin (90° - 0.5\,\Delta)} + R \sec 0.5\,\theta - R \quad \ldots (20)$$

Case 2. $\Phi \geqq 0.5\,\Delta$ (*Transitional Throughout*).—In Fig. 2(*b*), angle OPA = $90° + 0.5\,\Delta - \alpha$. Therefore,

$$T = \frac{c \sin (90° + 0.5\,\Delta - \alpha)}{\sin (90° - 0.5\,\Delta)} \quad \ldots (21a)$$

and

$$E = \frac{c \sin \alpha}{\sin (90° - 0.5\,\Delta)} \quad \ldots (21b)$$

9. Curve Control Data for Spiral

It is desirable to know the curve data for a transition spiral for which V, ρ, γ, and Δ are given. After computing Φ by Eq. 16*c*, such data may be found, for the two cases, in six steps, as follows:

Step	Case 1	Case 2
1	Compute R by Eq. 16*a*	Compute K by Eqs. 4, 16*a*, and 16*b*
2	Compute S by Eq. 3, 4, or 16*b*	Compute S by Eq. 3
3	Find α by referring to Section 5	Compute R by Eq. 4 or 16*b*
4	Find c by referring to Section 6	In Section 5, find α corresponding to $\phi = 0.5\Delta$
5	Compute $\theta = \Delta - 2\Phi$	Find c by referring to Section 6
6	Compute T by Eq. 18	Compute T by Eq. 21*a*

It must not be thought that there is only the one transition spiral suitable for a given velocity V and intersection angle Δ. On the contrary, because of the freedom of choice in selecting the limits ρ and γ, there are any number of permissible spirals. For example, in case 1, it is not necessary for the vehicle actually to achieve the maximum centrifugal ratio. By reducing ρ, the radius and the central angle of the central circular section are increased, and the lengths of the bounding spirals are reduced. In both cases γ can be reduced, thus increasing the total length S. As a matter of fact, in the three formulas, Eqs. 16, there are six variables, Φ, ρ, γ, V, R, and S. This means that any three of them may be chosen arbitrarily (within practical limits) and the other three thereby determined. In forming tables of curve data it may be considered convenient to fix, say, V, S, and R. For a given V and an accompanying variety of suitable values of S and R the designer would thus be able to give data for a number of allowable spirals for each degree of Δ. This has been done, for example, by Mr. Barnett (2*a*).

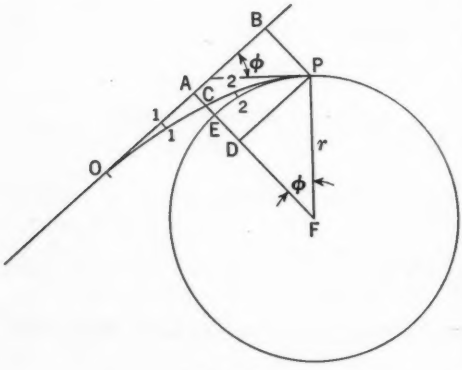


Fig. 3

10. Two Useful Relations

Let O (Fig. 3) be the origin and P any point of the spiral, and let F be the center of the osculating circle for P. Let the perpendicular FA from F to the tangent line OA cut the circle at point E and the spiral at point C. Denote length AE by σ and length OA by τ. Let D be the base of the perpendicular from P on FA and B the base of the perpendicular from P on OA (see Fig. 3). Then, $\sigma = \overline{BP} - ED = y - r(1 - \cos\phi) = K\phi^{1/2}\left(\frac{\phi}{3} - \frac{\phi^3}{42} + \cdots\right)$

$-r\left(\frac{\phi^2}{2}-\frac{\phi^4}{24}+\cdots\right)=s\left(\frac{s^2}{3K^2}-\frac{s^6}{42K^6}+\cdots\right)-\left(\frac{K^2}{2s}\right)\left(\frac{s^4}{2K^4}-\frac{s^8}{24K^8}+\cdots\right)$, since $\phi=\frac{s^2}{K^2}$ and $K^2=2rs$. Finally

$$\sigma=s\left(\frac{s^2}{12K^2}-\frac{s^6}{336K^6}+\cdots\right)\qquad(22a)$$

Similarly, $\tau=\overline{OB}-\overline{DP}=x-r\sin\phi=K\phi^{1/2}\left(1-\frac{\phi^2}{10}+\ldots\right)-r\left(\phi-\frac{\phi^2}{6}+\cdots\right)=s\left(1-\frac{s^4}{10K^4}+\cdots\right)-\frac{K^2}{2s}\left(\frac{s^2}{K^2}-\frac{s^4}{6K^4}+\cdots\right)$. Finally,

$$\tau=s\left(\frac{1}{2}-\frac{s^4}{60K^4}+\cdots\right)\qquad(22b)$$

11. Approximate Relations Good for Spirals of Small Deflection

There are a number of approximate relations that apply for small values of ϕ, and, within their ranges of negligible error, are extremely useful for both computation and field work. In this section, a few of these relations are developed. Nine such approximations are presented, as follows:

1. For $\phi<6°$, offsets from the initial tangent vary approximately as the cubes of the distances along the tangent;
2. Angle α is approximately equal to one third of ϕ; that is, $\alpha\approx\frac{\phi}{3}$;
3. Angle α varies approximately as the square of s;
4. For $\phi<10°$, the tangent distance τ varies approximately as $0.5\,x$, and therefore $\tau\approx 0.5\,x\approx 0.5\,s$;
5. For $\phi<10°$, the tangential offset AC, Fig. 3, is equal approximately to $0.5\,\sigma$;
6. When an upper limit ρ is developed, $\sigma\approx\frac{g^3\rho^3}{24\gamma^2}$, which is a constant;
7. The tangent distance T is equal approximately to $0.5\,S+(R+\sigma)\tan 0.5\,\Delta$;
8. The external radial distance E, from the point of intersection of two tangents to the curve, is approximately equal to $(R+\sigma)\sec 0.5\,\Delta-R$; and
9. Offset distances (1-1, Fig. 3) from the tangent to the spiral are approximately equal to the distance (2-2, Fig. 3) from the spiral to the tangent osculating circles, at equal distances from the opposite ends of a spiral.

Approximation 1.—For $\phi<6°$, offsets from the initial tangent vary approximately as the cubes of the distances along the tangent. By Eqs. 6, $y\approx\frac{1}{6}K\phi^{3/2}$ and $x\approx K\phi^{1/2}$. Eliminating ϕ:

$$y\approx\frac{x^3}{6K^2}\qquad(23)$$

Thus, for small deflection angles, the transition spiral is approximated by a

cubic parabola. The cubic parabola can be used successfully as an easement curve (6) for deflections of about 8.5°, because all cubic parabolas reach their maximum curvature at $\phi = \text{arc tan } \frac{\sqrt{5}}{15} \approx 8.5°$.

Approximation 2.—Eq. 8, Section 5, indicates that when $\phi = 15°$ the error in assuming $\alpha \approx \frac{\phi}{3}$ is about 10″ (see list of C-values in Section 5).

Approximation 3.—Angle α varies approximately as the square of s because $\phi = \frac{s^2}{K^2}$; and therefore $\alpha \approx \frac{s^2}{3\,K^2}$.

The formula for the lemniscate (see Fig. 4) is

$$c^2 = m^2 \sin 2\,\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

in which m is the value of c at the midpoint of the lobe. For this curve the angle α equals exactly $\frac{\phi}{3}$. A special study of the lemniscate was made by Mr. Royal-Dawson (22). It can be shown that for the lemniscate $c\,r$, and not $s\,r$,

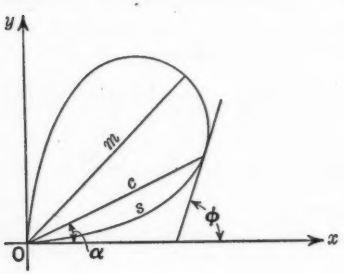


FIG. 4

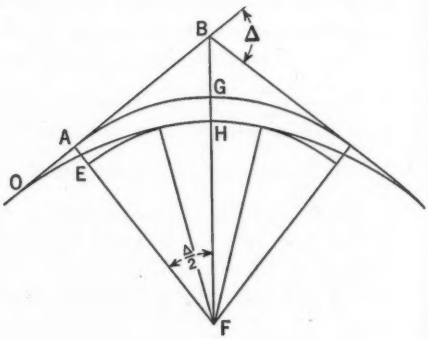


FIG. 5

is a constant. The objections to the lemniscate seem to be that s bears no simple relationship to any of the angles involved and hence the curve does not lend itself readily to calculations concerning s and r. Moreover, it is not a simple matter to lay out a lemniscate from a point on itself.

Approximation 4.—For $\phi < 10°$, the tangent distance $\tau \approx 0.5\,x \approx 0.5\,s$ as can be concluded from Eq. 22*b*.

Approximation 5.—For $\phi < 10°$, the distance $\text{AC} \approx 0.5\,\sigma$ because (see Section 10):

$$y = s\left(\frac{s^2}{3\,K^2} - \frac{s^6}{42\,K^6} + \cdots\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Therefore,

$$\overline{\text{AC}} = \frac{s}{2}\left(\frac{s^2}{12\,K^2} - \frac{s^6}{2{,}688\,K^6} + \cdots\right) \approx \frac{\sigma}{2} \quad \ldots\ldots\ldots\ldots\ldots (26)$$

Thus, by approximations 4 and 5, it is clear that s and σ bisect each other approximately when ϕ is small.

Approximation 6.—When ρ is developed, $\sigma \approx \frac{g^3 \rho^3}{24 \gamma^2}$ (which is a constant) because (see Eq. 26) $\frac{\sigma}{2} \approx \overline{AC} \approx \frac{\overline{BP}}{8} = \frac{y}{8}$. Therefore,

$$\sigma \approx \frac{y}{4} \approx \frac{s^3}{12 K^2} = \frac{s^2}{24 r} = \frac{g^3 \rho^3}{24 \gamma^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

Approximation 7.—From Fig. 5,

$$T = \overline{OA} + \overline{AB} = \tau + (R + \sigma) \tan 0.5 \Delta \approx 0.5 S + (R + \sigma) \tan 0.5 \Delta \ldots (28)$$

Approximation 8.—In Fig. 5, $\overline{BH} = E$, $\overline{HF} = R$, and $\overline{AE} = \sigma$. Then, by geometry:

$$E = (R + \sigma) \sec 0.5 \Delta - R \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Approximation 9.—In Fig. 3, let 1-1 be an offset from the tangent (perpendicular to the tangent) and let 2-2 be an offset from the osculating circle (normal to the osculating circle), and further assume that spiral arc O1 equals spiral arc P2. Then 1-1 $\approx$ 2-2, because, from Section 3, $\frac{d\phi}{ds} = k s$, whence $\frac{d^2\phi}{ds^2} = k$. Thus, the arc rate of change of the curvature for the spiral is constant; and the spiral departs from any osculating circle at the same arc rate as from the initial tangent. Hence, if ϕ is small, 1-1 $\approx$ 2-2. This property is convenient when using the method of deflection angles and the entire spiral cannot be set out from the origin, necessitating moving up along the spiral (21).

12. Staking the Transition Spiral

There are three field methods establishing intermediate points on the spiral—the method of deflection angles, the method of tangential offsets, and the method of middle ordinates. Of these the first is the one most generally used in highway work and this section will be devoted to a brief examination of the method. For the remaining two procedures the reader is referred to the work of the late G. W. Pickels and C. C. Wiley, Members, Am. Soc. C. E. (21).

The problem is to lay out the spiral by polar deflection angles and constant chord lengths. The constant chord length will be referred to as the "unit chord." This unit chord must approximate closely the corresponding arc length, and should therefore not exceed 0.1 R. Denoting the unit chord by s and its deflection angle by $\bar{\phi}$, since ϕ varies as s^2,

$$s = S \sqrt{\frac{\bar{\phi}}{\Phi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

To determine s, set $\bar{\phi}$ in Eq. 30 equal to 3′, 12′, 27′, or 48′, selecting that value which makes s of most suitable length. Having thus chosen s and $\bar{\alpha} = \bar{\phi}/3$, proceed to set out the spiral by polar deflection angles. In Table 1, the polar deflection angles have been computed by the fact that ϕ varies as

s^2 and that (see Eq. 8) $\alpha = \phi/3 - C$, and then recorded to the nearest 0° 0′ 20″, this being the limit of graduation of instruments in normal use (5).

It may be considered desirable to divide the spiral into a number of equal chords and then to set out the points from the formula,

$$\bar{\alpha} \approx \alpha \left(\frac{\bar{s}}{S}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

which is sufficiently accurate for values of ϕ to about 18°. Extensive tables, corrected for the approximation, have been prepared for this method (2), yielding the so-called ten-chord and twenty-chord spirals, ten or twenty usually being the adopted number of equal chords.

Both of these methods are open to the objection that the points fixed do not coincide with the even chain lengths along the curve; but in the absence of complete tables for the spiral the saving of time and computation is considered worth while.

TABLE 1.—POLAR DEFLECTION ANGLES FOR STAKING A SPIRAL BY CONSTANT CHORD LENGTHS

VALUES OF α:[a]				Polar deflection
0° 1′	0° 4′	0° 9′	0° 16′	
1	....	....	....	00° 01′ 00″
2	1	....	....	00° 04′ 00″
3	....	1	....	00° 09′ 00″
4	2	....	1	00° 16′ 00″
5	....	....	....	00° 25′ 00″
6	3	2	....	00° 36′ 00″
7	....	....	....	00° 49′ 00″
8	4	....	2	01° 04′ 00″
9	....	3	....	01° 21′ 00″
10	5	....	....	01° 40′ 00″
11	....	....	....	02° 01′ 00″
12	6	4	3	02° 24′ 00″
13	....	....	....	02° 49′ 00″
14	7	....	....	03° 16′ 00″
15	....	5	....	03° 45′ 00″
16	8	....	4	04° 16′ 00″
17	....	....	....	04° 48′ 40″
18	9	6	....	05° 23′ 40″
19	....	....	....	06° 00′ 40″
20	10	....	5	06° 39′ 40″
21	....	7	....	07° 20′ 20″
22	11	....	....	08° 03′ 20″
23	....	....	....	08° 48′ 00″
24	12	8	6	09° 34′ 40″
25	....	....	....	10° 23′ 40″
26	13	....	....	11° 14′ 00″
27	....	9	....	12° 06′ 40″
28	14	....	7	13° 01′ 00″
29	....	....	....	13° 57′ 20″
30	15	10	....	14° 55′ 20″
31	....	....	....	15° 55′ 20″
32	16	....	8	16° 57′ 20″
33	....	11	....	18° 01′ 00″
34	17	....	....	19° 06′ 20″

[a] Number of unit chord lengths from the beginning of the curve, for the values of α shown.

13. DEGREE OF CURVATURE

Without entering into the arguments for or against (23) the use of the idea of "degree of curvature," it remains a fact that it is the present American practice to treat curves in terms of this concept; hence a word or two will be offered in this section relating the equations of the previous sections to those involving the degree of curvature. Many definitions of degree of curvature exist, but that which seems most acceptable at present is the following (21):

Definition 1.—The degree of curvature, d, of a circular curve is the central angle of an arc of 100.007 ft.

Definition 2.—Extending Definition 1, the degree of curvature, d, of a transition spiral at any point is the degree of curvature of its osculating circle at that point. The final degree of curvature of the spiral is D.

Definition 3.—Let κ represent the rate of change of the degree of curvature of the transition spiral per 100 ft of arc length. The spiral can then be classified as a κ°-spiral (21). Relations connecting r and d, K and κ may be found as follows: The circumference of a circle equals $2\pi r = \frac{100.007 \times 360}{d}$; whence,

$$r = \frac{100.007 \times 360}{2\pi d} = \frac{5{,}730}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

From Eq. 4, $2rs = K^2$; and $\frac{2(5{,}730)s}{d} = K^2$. Making $s = 100$ ft and $d = \kappa$:

$$\kappa = \frac{200 \times 5{,}730}{K^2} = 100 \times 5{,}730\, k \quad \ldots\ldots\ldots\ldots\ldots\ldots (33)$$

By use of Eqs. 32 and 33, r (or R) and K can be substituted in any of the formulas of this paper by d (or D) and κ. Messrs. Pickels and Wiley (21) have presented a treatment along these lines.

14. Conclusion

One of the many problems encountered in highway engineering is that of curve construction and the consequent superelevation. The staking of the superelevation on unspiraled curves was a hit-or-miss affair, placing most of the burden on the motorist. In addition to speed, considerations of safety and comfort must also be treated attentively in curve design. The answer to the problem is a continuous spiral whenever that is possible. In determining when a curve may be transitional throughout it became necessary for the writers to examine the mathematical theory of the transition spiral. The first step was to examine the existing readily obtained literature, where it was found that the mathematical theory of the spiral was often obscured for one or more of the following reasons:

1. Although numerous formulas were offered, often no proof, or very little proof, was given;
2. Comfort and safety requirements were not sufficiently considered;
3. Generality was lost because field men were soon weaving into formulas specially adopted or favored values for certain parameters (for example, numerical values of ρ and γ might be adopted early in the treatment with the result that the significance of these parameters is lost forever in the sequel);
4. In an effort to be practical the essential mathematics was obscured by the mathematically extraneous idea of "degree of curvature";
5. Confusion and inconsistency were created by neglecting to make clear which mathematical properties of the curve are exact and which are approximate, or by neglecting to inform the reader to what extent a formula is approximate;
6. Often the mathematics was developed with the principal object of explaining or developing a particular set of tables, or the treatment was otherwise (purposefully, perhaps) one-sided; and

7. In a few cases (British), the treatment was too massive or exhaustive.

Perhaps this paper obviates some of these faults and will better serve the growing number of persons who need a clear and concise introduction to the fundamental principles of the curve.

Historically, the transition spiral was first investigated by the Swiss mathematician, Leonard Euler, and seems to be a mathematically rich curve. R. C. Archibald has presented an excellent historical sketch concerning the curve and its earlier applications to the study of elastic springs and to the theory of optics (35). An amazingly rich bibliography of the mathematical development of the properties of this curve is given. There is no duplication between this bibliography and the remaining items in the Appendix. In all likelihood the Euler curve possesses some still unknown properties (exact or approximate) which perhaps could be valuable to the engineer. Who knows, for example, how well the engineer might improve on the present osculating circle method of laying out the spiral when he faces the need for moving up along the curve (15)? The mathematical pole of the spiral can be found easily; can it be used? What new problems will the spiral present? Will there be any need for a rational method of applying vertical transition at the ends of the spiral? How can the engineer best devise such a method? Can he improve upon existing methods of describing right of ways for these curves (8)?

APPENDIX

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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REPORTS

WATER SUPPLY ENGINEERING

REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION FOR THE TWO YEARS ENDING DECEMBER 31, 1944

The last report of the Committee on Water Supply Engineering was for the two years ending December 31, 1942.[1] This report, as was that, is written in a period when the normal progress in water supply is upset, but in special ways stimulated, by the country's intensive devotion to war enterprises. The Committee is, as in the last report, including some statements of conditions for the record.

For many industrial plants water supplies have been increased, with purification plants built in considerable numbers. Existing water plants have been greatly enlarged where required to supply war industries; whereas, on the other hand, the normal course of improvements to water supplies for cities which have no marked increase in war industries has been interrupted. San Diego, Calif., the East Bay Municipal District (San Francisco Bay, Calif.), and Detroit, Mich., are striking examples of the former class. New York, N. Y., is the outstanding example of the latter class, since its huge new Rondout-Delaware supply has stood for nearly three years with its first dam only partly completed and only the natural flow of Rondout Creek to make use of its long new tunnel from the Catskill Mountains.

During 1944 optimism as to a quick victory in the war grew in official circles at Washington, D. C., and many limitation and conservation orders were revoked or revised. Cutbacks in manufacture of military material of 30%, 40%, and even 50% were planned by the War Production Board (W.P.B.) to follow the expected quick victory in Europe. Its regulation No. 25, the "Spot Authorization" plan, was put into effect. For utilities, including water, the limit for the material cost of plant additions requiring specific approval was raised from $1,500 to $10,000, where it still (March, 1945) stands. Early in 1944, water utilities were permitted for the first time in several years to use the copper tubing then in their inventory; later new copper tubing was made available. Bronze was again permitted for use in producing water meters in sizes up to 1 in. Because of the shortage of form lumber and an increased supply of steel plate fabrication and erection of steel water storage tanks were again made possible.

NOTE.—Please forward all comments on this report directly to Chairman Thomas H. Wiggin, 90 Broad St., New York 4, N. Y.

[1] *Proceedings*, Am. Soc. C. E., March, 1943, p. 399.

During 1944, however, even previous to the military delays in November and December in France and to a lesser extent in Italy, there was a progressive labor shortage due, in part, to the movement of labor to peacetime business in anticipation of the shutdown of the more lucrative war jobs. The "Area Production Urgency Committee" system was set up, first on the West Coast and then in other areas in the Middle West and East. As the situation became still worse, so-called "Modified Area Production Urgency Committees" were instituted in nearly all parts of the United States with power to pass on and recommend approval or rejection of production or construction projects requiring additional man power. Where at the beginning of the year the use of critical materials was of prime consideration, later additional construction could only be allowed if it could be demonstrated to these committees that man power was available. Following the military difficulties late in 1944 and the resumption of full speed in the manufacture of materiel of war, all construction projects again were closely scrutinized both with regard to critical materials and with regard to available labor.

It is still thought to be the opinion of the Water Utilities Branch of W. P. B. that water supplies are inadequately maintained for safety, that needed extensions should be made, and that less urgent but sound projects should be pushed where man power is available. While army and navy installations, except in conquered lands, are probably nearly complete, it has been stated that additional manufacturing plants are to be built at once and additional water facilities will doubtless be needed for them.

Production of Cast-Iron Pipe and Cement Lining Therefor

Information on tonnage or footage production during the years covered by this report—namely, 1943 and 1944—is not complete (see Table 1). It is obvious that the years 1943 and 1944 showed a very large reduction in orders

TABLE 1.—Cast-Iron Pipe Production and Interior Protections

Year	(*a*) Production[a] at Foundry:			(*b*) Standard Cement Lining[b] at Foundry:				(*c*) Thin Cement Lining[b] at Foundry:			(*d*) Tar[c] Coating
	B	C	D	B	C	D	E	B	C	E	E
1940	100	100	100	3.76	....	....	....[d]	8.86	....	....	....
1941	175	....[d]	....[d]	4.22	....	....	....[d]	8.32	....	....	....
1942	185	....[d]	....[d]	3.69	....	....	3.5	2.64	....	9.7	86.8
1943	65	58	40±	0.93	3.5	3±	5.3	8.53	31.2	17.2	77.5
1944	70	52	55±	5.35	4.9	10±	8.3	16.58	45.8	44.4	47.3

[a] In percentages of the 1940 data taken as 100%. Data for foundries A and E are not available. [b] Percentages of total output. At foundry A, data were not available in 1940, 1941, and 1942. In 1943 most of the orders at foundry A were for war plants with no time available for lining; and 25% of the total output of foundry A, in 1944, was for standard cement lining (Table 1(*b*)) with another 25% for thin cement lining (Table 1(*c*)). At foundry D some thin cement lining (Table 1(*c*)) was produced, but very little. [c] Data not available for foundries A, B, C, and D. [d] Data not available.

for cast-iron pipe as compared with 1941 and 1942 but that call for cement lining for normal plant extension work is stronger, particularly for the thin cement lining, sometimes called "Enameline," for which there is often no

direct charge. In the opinion of the Committee there is a question as to whether the thin lining provides protection equal to that furnished by the linings of standard thickness.

Reinforced Concrete Pressure Pipe

Reinforced concrete pressure pipe installations 30 in. to 60 in. in size, which in 1942 reached the very high peak of nearly 150 miles, fell off markedly in 1943, and much more in 1944.

Lining of Large Pipe in Place

The process of lining large pipe in place by a traveling, centrifugal, mortar-applying machine, with revolving paddles attached, has continued in the field. The company engaged in this work reports a total of 43 miles of 36 in. to 60 in. pipe lined in 1943 and 1944. At Rochester, N. Y., the old 36-in., No. 1 wrought iron conduit, laid in 1874 and abandoned in 1934, was lined with $\frac{3}{8}$ in. to $\frac{1}{2}$ in. of mortar and put back into service in December, 1944. This pipe was as much as 6 in. out of round, making cleaning difficult and smoothing of the mortar by machine practically impossible.

Lining of Small Pipe in Place

The American branch of the Australian process of applying cement lining by mandrel to pipe in place in diameters 3 in. to about 16 in. has continued active during the war. Up to the end of 1942, when the Committee's last report was made, about 50 miles of pipe had been so lined in North America. By the end of 1944 this total had increased to about 73 miles. In addition, about 4 miles of pipe had been lined in yards previous to laying.

Notable Water Supply Projects

The paramount needs for war have interrupted nearly all water supply projects except where increased supply was shown to the W. P. B. to be needed for increased supply to plants manufacturing war goods or goods closely related to the war; or for increased supply to railroads, mines, etc., directly concerned in the war.

The Tennessee Valley Authority has continued to build dams and hydro-electric power plants. The added power is being used by new or enlarged war manufacturing plants.

The Central Valley (California) Project was largely arrested for a time but resumption of construction on the huge Shasta Dam was soon authorized. The state of the work as of January 1, 1945, was as follows:

Shasta Dam was practically complete except for installation of spillway gates. Storage of water behind the dam commenced in January, 1944. Power generation began in July, 1944.

Friant Dam on the San Joaquin River was complete except for spillway and outlet gates. Valves borrowed from Boulder Dam have been installed on two of the four outlets to permit partial control of the reservoir.

No work had been done on Friant-Kern Canal extending to Bakersfield, Calif.

Contra Costa Canal had been partly completed and that part was in service.

San Diego, Calif.—Water supply requirements in the San Diego area have probably increased as rapidly as anywhere in the United States during the years of war activity. The city has recently completed the San Vicente Dam at a cost of approximately $3,275,000; important additions have been made to its distribution system consisting of mains, filter plants, and pumping plants; and a plan is being made to obtain additional water from the Metropolitan Water District of Southern California. To obtain this outside supply, the city, together with a large adjacent area, has formed what is called the San Diego County Water Authority. This Authority, working with the U. S. Navy Department, the Federal Works Agency, and the Bureau of Reclamation, has conducted investigations to determine the best method of procedure. At present negotiations are under way for annexation of this Authority to the Metropolitan Water District of Southern California.

Arrangements have been completed by which the Navy Department will finance the construction of an aqueduct from the west portal of the San Jacinto Tunnel on the Metropolitan Aqueduct to the San Vicente Reservoir. This aqueduct will have a length of approximately 72 miles and is estimated to cost some $18,000,000. The Bureau of Reclamation will provide the engineering plans and specifications, and the Navy Bureau of Yards and Docks will do the construction work. The cost of the project is to be shared by the War Department, Navy Department, and Federal Works Agency. Under this plan, upon the termination of the war, a contract will be negotiated between the Navy Department and the San Diego County Water Authority for the purchase of the aqueduct from the Navy. This aqueduct will have a capacity of 77.5 cu ft per sec. It is planned to begin construction about April, 1945. The municipalities who have shared in the cost of the Colorado River supply of the Metropolitan District are exercised because no provision has yet been made by the federal government for sharing on behalf of San Diego the cost of producing the supply from the Colorado River.

Los Angeles, Calif.—The City of Los Angeles has also had a very substantial increase in water consumption and is now utilizing the additional supply obtained from Mono Basin, which is being transmitted to the city through the Owens Valley Aqueduct. With this supply, Los Angeles, as yet, has not had to purchase any very large quantity of water from the Metropolitan Water District. Quite a number of additional facilities are being planned by the City, among which is the Baldwin Hills distribution reservoir proposed for early construction, with a capacity of approximately 300 million gal.

Metropolitan District of Southern California.—The Metropolitan Water District of Southern California is now supplying water through its La Verne Softening and Filtration Plant to all but two of the member cities. The plant has a capacity of 100 mgd and during the past year the output has reached a maximum of about 45 mgd. With the continued war activities in Southern California, the output undoubtedly will be increased materially during the

next year. The plant has already been of great value to the various Southern California cities in providing an ample supply during the present war crisis.

San Francisco, Calif.—The City of San Francisco has been engaged on constructing an additional outlet tunnel at San Andreas Reservoir. Plans are under way for the installation of an additional pipe line from that reservoir to the city as soon as certain priorities can be obtained permitting the purchase of the necessary steel.

East Bay Municipal District.—Water consumption in the East Bay Municipal Utility District (Oakland, Calif., and vicinity) has increased steadily and rapidly from about 43 mgd in 1938 to 86 mgd in 1944, and is expected to reach 94 mgd in 1945. This increase has been due entirely to the extensive war activities in the East Bay area.

During 1944 approximately 26.9 miles of cast-iron pipe ranging from 4 in. to 24 in. have been installed to improve distribution facilities and also to extend service to new housing projects and to war industries. Six miles of 36-in. steel line, cement lined and gunited, were installed between the San Pablo Filter Plant and Richmond, Calif., in order to improve the supply for the Richmond shipyards, the Standard Oil Refinery, the many housing projects, the Mare Island Navy Yard, and numerous other industries in that vicinity. During 1944 also the District has been supplying approximately 6 mgd to Vallejo, Calif., for use in that city and in the Mare Island Navy Yard. Due to increased activities, these requirements are being stepped up to 9 mgd and the District has under construction an additional pumping plant on the pipe line between Richmond and Vallejo, together with some 3,000 ft of 25-in. steel pipe in the vicinity of Crockett, Calif. This installation is expected to be complete and available for use early in 1945.

Contracts have been let for the construction of a second, or Bixler, pumping plant on the Mokelumne Aqueduct, which aqueduct transmits water from the Pardee Reservoir to the District, a distance of some 93 miles. This is the District's principal source of supply. The aqueduct has a capacity of some 67 mgd with the Walnut Creek pumping plant in operation. By the construction of the Bixler pumping plant, located approximately 33 miles east of, and operating in series with, the Walnut Creek plant, the capacity of the aqueduct will be increased to 95 mgd. The Bixler plant will have two vertical-type pumps of 47½ mgd capacity each, operated by 4,000-hp motors, and will pump against a maximum head of approximately 450 ft. This plant is being constructed in cooperation with the Federal Works Agency.

Tacoma and Seattle, Wash.—The City of Tacoma is replacing some 10 miles of 60-in. wood-stave pipe with 60-in. pre-cast concrete pipe, and Seattle is replacing some 2 miles of 60-in. wood-stave pipe with 60-in. steel pipe. These replacements will insure a continuous supply in those important centers of war activity.

Salt Lake City, Utah.—At Salt Lake City, an additional water supply is to be provided through the Deer Creek Aqueduct under a cooperative agreement between the City and the Bureau of Reclamation. The Bureau has under construction all of the concrete sections of the aqueduct, which make up about three fourths the length. The remaining one fourth is to be constructed of

steel pipe and this section is being held up by the W. P. B. with the understanding that in case of any threatened water shortage, sufficient steel will be released to permit its installation.

Chicago, Ill.—Construction work on Chicago's South District Filtration Plant has progressed to the extent that the huge plant is about 70% completed, as of December, 1944. The plant is to be operated partially in 1945. The water will be chlorinated, coagulated, and settled; but not filtered until priorities on the filter equipment are obtained and the equipment is installed.

Baton Rouge, La.—A plant of 20,000 gal per min. capacity for clarifying Mississippi River water without filtration was constructed at Baton Rouge for use in certain industrial plants requiring cooling water. The Spaulding principle was employed. The water is for the purpose of replacing, during the months when the Mississippi River is cool enough, water from a much overpumped well field.

Federal River Projects

In the midst of war the administration has urged and Congress has passed bills which seem to be designed to have a far-reaching effect on the development of rivers for all purposes and which necessarily affect water supply. House Bill 4485 authorizes the construction of public works on rivers for flood control, navigation, irrigation, and park purposes with provisions for allowing use for water supply. This work, except as noted subsequently, is assigned to the Corps of Engineers with the proviso that the Reclamation Service shall be kept informed and, to the extent deemed practicable by the Chief of Engineers, permitted to participate in investigating works west of the 97th meridian (which lies just west of Minnesota and Iowa and in the easterly parts of Nebraska, Kansas, Oklahoma, and Texas); also that the states shall be consulted and, to the extent deemed practicable by the Chief of Engineers, allowed to participate in investigating works within their borders. Reports and recommendations on projects are to be printed as House or Senate documents.

The use for navigation of any waters west of the 98th meridian is to be only such as shall not conflict with present or prospective use for "domestic, municipal, stock water, irrigation, mining, or industrial purposes."

Flood control is placed under the War Department but federal investigations of watersheds and measures for runoff and water-flow retardation and soil-erosion prevention on watersheds are assigned to the Department of Agriculture. No reference is made to stream gagings, now under the Department of the Interior.

Parks may be constructed by the War Department or under license by it. They shall be open for such boating, swimming, bathing, fishing, etc., as the War Department considers not contrary to the public interest.

Surplus hydroelectric power generated by plants built by the War Department shall be delivered to the Secretary of the Interior for disposal under arrangements affording lowest rates to the people. The Secretary of War is empowered to make contracts with states, municipalities, private concerns, or individuals for domestic and industrial uses of surplus water, provided that

"no contracts for such water shall adversely affect then existing lawful uses of such water." If the Secretary of War, upon recommendation of the Secretary of the Interior, shall determine that any dam and reservoir project operated under direction of the Secretary of War may be used for irrigation, the Secretary of Interior may construct additional works for their utilization provided such works shall be self-supporting under the Reclamation Service law.

The bill provides for improvements on the Missouri River to be constructed by the War Department and Reclamation Service, but the President, in signing the bill, served notice that he would later ask for a Missouri River Authority similar to the Tennessee Valley Authority (TVA). Such a bill, H.R. 5410, was introduced on September 20, 1944, but not then passed.

House Bill 4485 authorizes work and expenditure on the planning of these projects during the war for the purpose of preparing for prompt postwar construction.

To list the rivers affected by this bill would take too much space. The river basins in which these projects lie are as follows: Lake Champlain, N. Y., Basin; Blackstone River, Mass.; Connecticut, Thames, and Hoosatonic rivers, Conn.; Susquehanna River Basin; Roanoke River Basin, Va.; Edisto River Basin, South Carolina; Savannah River Basin, Georgia; Mobile and Alabama, Coosa River Basins; Lower and Upper Mississippi River Basins; Red-Ouachita River Basin; Arkansas River Basin; White River Basin; Red River of the North Basin; Missouri River Basin; Ohio River Basin; Great Lakes Basin; Colorado River Basin; Bragos River Basin; Rio Grande River Basin; Great Salt Basin; San Diego River Basin; Ventura River Basin; Santa Ana River Basin; Los Angeles-San Gabriel Basin and Ballona Creek; Pajara River Basin; Sacramento-San Joaquin River Basins; Napa River Basin, California; Coquille River Basin, Oregon; Nehalen River Basin, Oregon; Willamette River Basin, Oregon; Columbia River Basin, Oregon; Willapa River Basin, Washington; Chehalis River Basin, Washington; and Territory of Hawaii.

In addition, 43 individual projects which appear in some cases already included broadly in "basins" are listed separately for investigation.

There is included also a list of about a dozen watersheds upon which erosion prevention studies are particularly authorized during the war and prosecution of remedies ordered to be begun not later than immediately following the cessation of hostilities. Acquirement of land for these projects is made subject to the legislatures of the states in which the projects lie and a provision is made to pay to counties 1% annually of the acquisition price or valuation of the lands acquired.

The total appropriations for the projects in this bill amount to more than one billion dollars. It is not stated what part of the total estimated cost this represents. It is customary for Congress to make partial appropriations and for projects to be modified as studies progress. In many cases construction is apparently authorized in accordance with reports from the Chief of Engineers received in previous times, yet participation by states and the Reclamation Service in "investigations" is emphasized and there is a preamble which states

> "It is hereby the policy of the Congress to recognize the interests and rights of the States in determining the developments of the watersheds within their borders and likewise their interests and rights in Water Utilization and Control, as herein authorized to preserve and protect to the fullest possible extent established and potential uses, for all purposes, of the waters of the Nation's rivers; to facilitate the consideration of projects on the basis of comprehensive and co-ordinated development; and to limit the authorization and construction of navigation works to those in which a substantial benefit to navigation will be realized therefrom and which can be operated consistently with appropriate and economic use of the waters of such rivers by other users."

The Committee believes that this bill is the greatest step ever taken in the United States to nationalize and incidentally militarize river and water supply engineering. River work which is likely to take a generation or more has been assigned to the War Department by Congress and is already being diligently pursued by organizations assembled for war construction but now released in considerable part by completion of most of the war installations. Civilian engineers may share in this work as subordinates in the U. S. District Engineer offices and perhaps sometimes by taking over in their consulting offices some problem assigned to them by the Chief of Engineers.

The Committee believes that the purpose of coordinated development of rivers expressed in the bill is a proper one but its pursuit on such a huge scale in the spirit of wartime haste is questionable. There is also a question which should interest the Society as a whole, and the Sanitary Engineering Division in particular, because of its interest in the use of rivers and reservoirs—namely, the question of whether a broader organization with opportunity for any engineer, no matter where educated, to reach a position of authority and recognition, should not be created to handle this huge river work.

Postwar Planning

House Bill 4485 previously discussed, also House Bill 3961 (authorizing the improvement of a great many harbor and navigable rivers when appropriations shall be made, but not generally before six months after the cessation of hostilities) constitute the main contributions of the federal government to postwar activities related to water supply. States and municipalities have been and are likewise busy in listing potential improvements including water supply prominently. Appropriations have been made by some states to provide grants in aid of municipal planning for postwar work.

Postwar planning in the water supply field has received special aid through a committee organized in the summer of 1943 and sponsored by the American Water Works Association, New England Water Works Association, Sewage Works Federation, and the Water and Sewage Works Manufacturers Association.

For nine months, Field Director E. L. Filby, M. Am. Soc. C. E., on leave from his regular consulting work, gave his full time to promoting the development of plans and specifications for necessary and desirable water-works improvements and extensions. This work was aided by the state sanitary

engineers and by manufacturers' representatives. Special promotional material was provided.

The Committee on Post War Construction of the Society has also been very active, publishing a monthly bulletin on engineering construction plans under way. The bulletin dated December 1, 1944, gave the water-works projects as totaling $337,000,000 with an increase of $47,000,000 since the November 1 report was issued. These reports are based on statistics gathered and furnished by *Engineering News-Record.*

In a national inventory of needs for public water supply by H. W. Streeter, M. Am. Soc. C. E., and Ray Raneri, Assoc. M. Am. Soc. C. E., Engineers, U. S. Public Health Service, it is estimated that an annual cost of about $24,000,000 is paid by some 27,000,000 people who use water exceeding 90 ppm hardness that would be saved if the water were softened. It is also estimated that, if new water systems to supply small communities or groups without water supplies were installed, they would number approximately 5,000 and cost an estimated $180,000,000. If extensions and improvements were made in 6,500 communities at an estimated cost of $502,000,000 and $120,000,000 were spent for softening water supplies, there would be a requirement in the water supply field for slightly more than $800,000,000.

The Water Supply Committee is not convinced that all these public works will be needed in accelerated tempo and hopes that they will not be needed as they involve large increases in public debt. The preparatory work is very desirable, however, as definite plans would save waste if the system of made work should again be adopted. Doubts are caused by the fact that the need for the stimulus sought by pump priming was not felt for about 12 years after the cessation of hostilities in the First World War. Consumer goods and normal public works expansion may fill the needs for employment after this war; and, if they do not wholly suffice, there is still the question whether a successful method of pump priming can be devised.

Protection from Sabotage

At the time of the last report of this Committee, in January, 1943, means of protecting water supplies from sabotage had been developed and were being rapidly installed—such as watchmen, lights, and barbed-wire enclosures. There has been some reduction in the number of watchmen but the lights and wires remain. Provisions against bombing, such as camouflage and barriers, have largely passed from mind as the war has been definitely carried into enemy territory. Bombing of coastal cities by self-propelled bombs is believed by some naval men to be a possibility, but no one has suggested that such bombing could be directed at dams and aqueducts.

In the spring of 1943, English air forces bombed two of the German masonry dams, releasing the greater part of their stored waters. This was very carefully planned low-level bombing of the suicide type. Subsequently, one of the Italian dams was partially destroyed by bombing. These cases very materially altered the military point of view regarding possible damage to large dams.

Cross Connections for Mutual Aid

Taking a lesson from English experience and advice, state organizations for the care of water supply in war have promoted cross connections between adjacent water supplies; state laws have permitted and encouraged them for the duration of the war and a good many have been installed. These have had the double purpose of affording aid in case of sabotage and of reducing the amount of construction work and critical material required to meet new war demands for water. This latter effect has been accomplished where one supply has a surplus and an adjacent supply has a deficiency.

Sanitary Problems of Cross Connections where Non-Potable Supplies are Involved

In the East for many years it has been the general policy of state sanitary authorities to forbid cross connections between non-potable, industrial water supplies and potable supplies. Much attention has been given to ferreting out and abolishing such connections even against the plea of fire insurance engineers that a double check with air gap between removes all danger.

On the Pacific Coast the war has brought a special problem in cross connections in the protection of ships and docks. The Department of Public Health of the State of California took cognizance of this problem early in 1944 by informing all the coastal cities that recommendations were being made to the Public Health Service to grant only provisional certification of their water supplies because of the prevalent practice of interconnecting by means of fire hose the fire system on board ships with the dock hydrant system. Numerous cases of contamination had occurred when fire pumps of the vessels were used. The investigations by water supply officials resulted in the adoption of protective installations consisting of two gate valves, two check valves, and drains for testing the watertightness of the valves, these being placed as near the hydrant as practicable on the water system side. In addition, many water companies required the installation of a similar assembly at the point of service connection, generally as near the meter as possible.

In Southern California, particularly, another type of backflow prevention devices has been used to some extent instead of the standard check-valve installation. Briefly, these devices contain two check valves which are closed by hydraulic pressure whenever the pressure on the consumer's side rises above that in the public mains. In addition, a pressure differential relief valve is arranged so that the pressure in the chamber between the two checks drops to atmospheric whenever the pressure on the consumer's side rises above that in the public water mains. In case the check valve on the consumer's side leaks, the water which passes this check is discharged to atmosphere. Also, a vacuum relief valve on the water department side of the two checks breaks any vacuum which may occur in the public water mains. The back siphonage of unsafe water from the consumer's premises is thus prevented. These new devices are supposed to give more positive protection against backflow than the standard, double, check-valve installations. However, they are considerably more expensive to install and to maintain and in most cases

have not been in service long enough or in sufficient numbers to prove their worth.

The interest in cross connections has increased considerably in California during the last year, not only because of the matter of water certification, but also because of the new cross-connection regulations adopted by the State Department of Health in August, 1943. In the East Bay area, the policy has been adopted recently of requiring standard check-valve installations to be made by the consumer on all service lines supplying premises with another source of water supply. This policy was adopted because past experience indicated that rules and regulations against cross connections were not effective. For example, in one resurvey of industrial plants in the East Bay area, it was found that 21% of the plants made new cross connections within five years after statements had been signed by the consumer to the effect that all cross connections had been eliminated.

The Cross-Connection Regulations of the California State Department of Public Health permit cross connections of certain types, providing approved double check-valve assemblies are installed to protect the public water system. Other types of cross connections—for example, cross connections within sewage plants—are positively prohibited.

Unusual Floods, Hurricanes, Droughts

Reference is made to the very complete information on rainfall, runoff, floods, and droughts in the publication of the U. S. Geological Survey entitled "Water Resources Review—United States and Canada," published monthly. A yearly summary for the water year is published following September of each year.

In the water year 1943 the water situation was generally favorable, rainfall being deficient in only 10% of the area of the country. Droughts occurred in Maine and Florida and several southern and southwestern states. A major, widespread, record-breaking flood occurred in an area of 200,000 sq miles covering seven states from Oklahoma to Michigan. The historic flood of 1933 was topped and in Indiana and Illinois the flood of 1913 was equaled or exceeded. In West Virginia a flash flood in which rainfall reached 15 in. in 2 hr caused loss of life to 23 persons.

Unusually long-continued high runoff occurred in the United States central area from Kansas north into Canada, beginning in 1943, becoming particularly heavy about April, 1944, and persisting throughout 1944. The U. S. Geological Survey chart for the water year ending September 30, 1944, shows this area covering parts of nine states and Manitoba, with record yearly stream flows of 270% and 320% of normal for western Iowa and eastern Nebraska, respectively.

A summer flood in Missouri in 1944 gave runoffs of 800 cu ft per sec to 1,500 cu ft per sec per sq mile on watersheds of from 5 to 25 sq miles. One in Virginia in September gave rainfall reported up to 17 in. and sent the James River to the greatest height known in the 45 years of record.

A hurricane, somewhat similar to, but generally less severe than, the 1938 hurricane, although reported in places as having the highest wind intensities

in 50 years, struck the New Jersey coast on September 14, 1944. As a result, some twelve relatively small communities along the coast were without public water supply for periods varying from approximately an hour to two days. The failure of the electric power supply was the most general cause of the lack of water. In one community, the engine room was flooded. At two places, the storage battery for starting an auxiliary engine was so run down that the engine could not be started. In one instance, the lack of cooling water prevented the operation of a gas engine. At one seaside resort, broken boardwalk hydrants interrupted water supply for some six hours.

When the hurricane hit the New England area it was generally less severe than the 1938 hurricane, and furthermore the lessons of the 1938 hurricane had resulted in the water supply systems generally having auxiliary sources of power. Reports indicated little interference there with the water supply as a result of this storm.

During the latter part of 1943 and nearly all of 1944, stream flow was below normal in eastern Washington, southern Arizona, and in the Ohio River valley, particularly the northern portion of the latter. Referring to the annual stream flow chart prepared by U. S. Geological Survey for the water year ending September 30, 1944, stream flow of 42% of normal is given in eastern Washington, the lowest on record for that location, and 55% for Ohio and southern Arizona.

Boston, Mass., noted a forty-four day local drought in June, 1944, with only 0.25 in. of rain. This equaled the previous dry record of May, 1926.

Dams and Spillways

The Committee has noted no significant change since its report[2] for the 10 years ending 1939 in the theory or construction of either masonry or earth dams. The consideration of capacity of spillway continues to receive attention, probably because of better information on floods and seemingly larger flood runoffs. This matter was treated in the Committee's report for the years 1940, 1941, and 1942.[1] The practice of the Corps of Engineers will doubtless be increasingly controlling. Their rule in the recent past is understood to provide for a flood in cubic feet per second of 5,000 times the square root of the area (in square miles) of the watershed; furthermore, they start with a full reservoir and add a factor of safety, such as 25%.

There is a growing tendency also to try and predict maximum rainfall, considering possibilities as to air saturation and air currents that might occur in combination in the given region. It is understood that such predictions have been exceeded in a few cases.

The general application of these very conservative methods to existing dams would afford a postwar budget of large proportions.

Ground-Water Supplies

The depletion of ground waters in various parts of the United States has been accelerated by the war demand. Generally, no adequate control exists

[2] *Transactions*, Am. Soc. C. E., Vol. 105 (1940), p. 1740.

over the withdrawal of such water. In 1943 the draft on the ground water supply in Louisville, Ky., was so great as to make it doubtful if the ground water would furnish the required water for refrigeration purposes during the 1944 summer season. As this water was urgently needed for war plant use, the city furnished cool, filtered water for recharging the wells and this water was recaptured by the draft from the ground water during the summer period when the surface water was too warm to be used for cooling and refrigerating purposes.

Reference has already been made to difficulties at Baton Rouge. A 30-mgd sedimentation plant was constructed to supply clarified Mississippi water for cooling purposes during the cool months in order to relieve the overdraft on ground water. Similar relief of ground-water draft has been practiced in Cincinnati, Ohio.

The use of gravel-packed wells with brass or bronze or concrete screens, sunk by the rotary or the bailing method, is steadily increasing. These are used where the yield is plentiful, because of less cost per unit and advantage of fewer, larger, and more economically-operated turbine pumping units. In some locations where salt-water intrusion occurs, the smaller wells for "skimming" the fresh water are found more desirable.

A more economical method of installing water level wells, using drive pipe only $\frac{1}{4}$ in. or $\frac{3}{8}$ in. in diameter, has been devised and used by engineers of the Soil Conservation Service in California.[3]

Progress and Trends in Water Purification

Supply of Chemicals.—Water works are experiencing no serious difficulty in obtaining essential chemicals for water treatment. Better raw material and improved methods of manufacture are enabling manufacturers of aluminum sulfate to offer products practically equivalent to the prewar material. There has been expansion of the idea of using silica for the improvement of coagulation and an increasing use of clay is reported as an aid for the coagulation process.

Break-Point Chlorination.—Significant developments have taken place in the chlorination of water. Treatment to the "break point" is being rapidly extended as a result of a better understanding of the advantages of active chlorine. Generally speaking, there has been improvement both in quality and taste by chlorinating to such an extent that there is chemical reaction between the chlorine and interfering compounds in the water. The rôle of ammonia, as it affects chlorination, is better understood. Often the effectiveness of chlorine, when applied in marginal doses, is reduced by the presence of ammonia and similar compounds. Break-point chlorination is the addition of chlorine in quantities sufficient to cause reaction between chlorine and the ammonia and other compounds present in the water. The term implies leaving in the water a sufficient amount of active chlorine for rapid germicidal action. The development of the flash test for residual chlorine and the arsenite test for false residual gives very valuable aid in the control of chlorination.

[3] *Western Construction News*, November, 1944, p. 77.

Chlorine Dioxide.—Recently, encouraging results have been obtained in the removal of taste and odor with chlorine dioxide. The development of relatively cheap means of producing this compound by reacting sodium chlorite with chlorine gives promise of its use to a considerable extent for improving the palatability of water. The treatment is claimed to be particularly effective in destroying phenolic compounds. Since relatively large amounts of activated carbon are required to remove phenol, chlorine dioxide may prove very helpful in phenol contaminated waters. The use of chlorine dioxide is being tried in a number of plants and a better appraisal of the method may be expected in the comparatively near future.

Industrial Purification Plants.—The huge industrial developments, which have taken place in connection with the war activity, have necessitated the construction of many new installations for the purification of water for industrial use. This has involved intense activity on the part of water-works equipment concerns, and many of these plants incorporate proprietary devices such as up-flow sludge blanket equipment. There has been a rapid development of the "package" type water purification plant which can be installed in units, capable of meeting varying loads. The same type of water-works equipment has been used to a large extent in camps and war areas where existing water supply facilities were inadequate.

Likewise, a rapid expansion has taken place in facilities for the softening of water to be used in industrial process work, particularly in the manufacture of synthetic rubber. In many cases, partial softening has proved adequate for industrial water, especially that intended for cooling purposes. Both the exchange process and the lime-soda process have been utilized extensively in these developments and, in many cases, partial softening has been accomplished by the use of lime alone. This field offers unlimited possibilities for the newer control devices, many of which utilize electrical principles.

Salvage of Lime.—Lime recovery at water softening plants is receiving attention and the time may not be far distant when some of the plants will not only be producing sufficient lime for the plant but perhaps an excess that may be offered for sale. Use of the centrifuge for concentrating sludge and for improving the quality of the precipitate indicates a future trend in the disposal of softening plant precipitates.

Basic-Exchange Plants.—The various procedures for the complete deionization of water, by passage through beds of ion exchange materials, appear to be practical for certain industrial applications and, to a limited extent, to larger water supplies. Generally, this process is too expensive for the softening of domestic water supplies where hardness and salinity are high.

High-Pressure Aeration and Chlorination.—A unique demonstration of the possibility of removing taste and odor from water by means of aeration is reported from West Virginia. The use of high-pressure nozzles combined with chlorine treatment and activated carbon, in successive steps, appears quite successful in the removal of objectionable taste and odor, caused by organic chemical wastes.

Drinking Water Standard of U. S. Public Health Service.—The 1942 drinking water standards adopted by the U. S. Public Health Service for use on interstate carriers developed a very extensive criticism from water-works operators. The validity of the objections raised was recognized by the Surgeon General—as a result an extensive interpretive letter was issued in reference to these standards under date of September 1, 1944.

Census of Water Treatment Plants.—The U. S. Public Health Service completed the first census taken of water treatment plants in the continental United States covering 5,372 plants. This was published at the beginning of 1943.

Fluoride Treatment.—Long-term experiments under way in New York State and in Canada, on the effect of additions of fluoride to water supplies for the prevention of dental caries, represent a unique undertaking in the field of mass medication. It is too early to predict any results.

Diatomaceous Earth Filters.—An interesting item in the furnishing of water supply for the field troops is the use of diatomaceous earth filter as a substitute for the more conventional sand filter. This filter is said to have advantages in the quality of water produced and in the capacity of the filter per unit of weight and per unit of volume. One important advantage of the diatomite filter is the removal of amoebic cysts which it had been found would pass through the sand filters. The diatomite is mixed with the water and has to be supported on some base. Cylindrical elements made of porous refractory material and helically wound wire have been found most adaptable, according to Army investigations.

Field units developed are reported to be of two sizes. One has 3.6 sq ft of filter area with a normal capacity of 15 gal per min, and weighs about 50 lb. The other has 10 sq ft of filter area, a normal capacity of 50 gal per min, and weighs about 350 lb. The filter unit includes the slurry feeder, with necessary valves and fittings for attaching a hose and pump, the pump being a separate item. The pump and engine weigh about 60 lb. The smaller unit is expected to take the place of the portable sand filter and treatment section, weighing together some 800 lb and rated at 10 gal per min. The larger unit is expected to replace the sand filter unit, rated at about 75 gal per min and weighing some 4,900 lb on skids.

Control of Corrosion.—The control of the corrosion of metals by water continues to receive the study it deserves and apparently considerable progress is being made in this field. At the Fall Meeting of the American Chemical Society a symposium was conducted on corrosion control in industry and information was presented on the effectiveness of polyphosphates, chromates, glucosides, and sodium silicate, as well as the more common alkalis. The methods of corrosion control in the water supply systems of camps and cantonments throughout the country were described. Approaching the same subject from another angle, considerable study is being given to the application of plastics and other materials for use as protective coatings on metal surfaces exposed to the corrosive action of water. The importance of this problem to water supply plants cannot be overemphasized.

Valuation of Public Utilities

The past two years has seen the completion of a report (1943) and a supplementary report (1944) by the Depreciation Committee of the National Association of Railroad and Utility Commissioners (commonly abbreviated to NARUC). These reports unequivocally recommended straight-line age-life depreciation based on original cost. The reports were opposed by a committee of the Society, a committee of the American Water Works Association, and several utility groups. In general opponents disagreed with the straight-line method and with the recommendation to correct by charges to surplus the deficiencies in depreciation reserve resulting from sinking fund or observed depreciation methods which had been approved by commissions and courts up to comparatively recent years.

The NARUC depreciation reports were discussed at a meeting of the NARUC body in November, 1944, and failed of approval by a rather narrow margin. A substitute motion to lay the reports over with privilege of amending was passed. There may have been some element of states rights in the vote as the membership of NARUC Depreciation Committee was to a considerable extent from federal commissions. There is now a period of perhaps eight months in which those who would amend the reports may get their ideas into concrete proposals as distinguished from criticisms.

The NARUC Depreciation Committee felt itself on firm ground on account of recent decisions of the U. S. Supreme Court, particularly that in the Hope Natural Gas Company Case. The decision in this case, handed down on January 3, 1944, is generally regarded as marking a turning point in regulatory practice, although there is a tendency on the part of some lawyers to wait for a confirming case where the utility might be financially hurt by a similar decision, which in this case was not true.

The Hope Natural Gas Case came to the Supreme Court on an appeal by the Federal Power Commission from a ruling by the U. S. Circuit Court of Appeals, which had set aside a Federal Power Commission order—requiring the Hope Natural Gas Company to make a substantial cut in its wholesale rates—because of failure by the Power Commission to consider certain elements of fair value. The Power Commission had predicated its order on a $6\frac{1}{2}\%$ return on a rate base of "actual legitimate cost," and the company had contended for an 8% return on a reproduction cost, 100% greater than the actual cost used by the Power Commission. The Supreme Court reversed the ruling of the Circuit Court of Appeals by a 5–3 decision, which is highly significant in point of principles stated.

Thus, the Court decided that henceforth reviews, under claims of confiscation, will not be allowed merely because a regulatory commission has used a wrong method, nor without demonstration by the utility "that the total effect," or "end result" will seriously affect its financial integrity.

The Court did not directly overrule the finding in Smyth vs. Ames that the fixing of nonconfiscatory rates requires consideration of a number of cost and value elements, but rather held that regulatory tribunals should not be bound by any formula so far as the issue of confiscation in federal courts is concerned.

This retreat from the judicial review, which has been available to utilities since the Smyth vs. Ames Case (1898), obviously leaves unprecedented power in the hands of regulatory commissions. The immediate effect on public utilities will depend largely on their location, as U. S. Supreme Court decisions are not binding in states where statutes govern commission proceedings in fixing intrastate rates. At present, the written law in about one half the states permits commissions to use their own methods of determining the rate base; in nine states the commissions must find "fair" and "reasonable" values; in nine states the commissions are required by law to consider reproduction cost, but not as the sole yardstick of value.

The Hope Case, however, expressly reverses the ruling in United Railways & Electric Co. vs. West (1930) to the effect that annual depreciation and rate-base value should be predicated upon a consideration of reproduction cost. Thus, apparently federal commissions and state regulatory commissions—not otherwise controlled by state law—may in future disregard reproduction cost.

Miscellaneous Information in the Water-Works Field

Water Supply of the Armed Forces in Africa.[4]—The water supply in North Africa was nearly all from wells having a high concentration of salts. The City of Oran, with a population of about 400,000, uses water with a sodium chloride content of 3,000 ppm to 4,000 ppm and a hardness of about 1,350 ppm. Tests showed that American troops could drink water with a salt content up to 1,500 ppm. Water was scarce and it was found necessary to place the following limits on its use:

Item	Kind of use	Fresh-water allowance (gal per capita per day)
	Drinking and Cooking:	
a	Hospitals	2
b	All other users	1
	All Purposes:	
c	Hospitals	5
d	All other users	2
	Washing and Showers:	
e	Hospitals	5
f	All other users	4
g	In cases where water-borne sewerage was approved	25

After World War I, similar conditions were found many places in France, except that salt water was not used.

Importance of Leakage from Water Systems.—Dewey Johnson estimates that the 14,500 public water supplies pump needlessly each year about 4 billion gallons of water daily at an estimated annual cost of $170,000,000.

Wire Wound Standpipes.—A process of pre-stressing concrete in a standpipe by tight, machine-winding of wire and subsequent protection of the wire by an exterior coat of mortar has been used. This appears to be the same principle as in wire-wound concrete pipe.

[4] From information supplied by William E. Stanley, M. Am. Soc. C. E.

Powdered Activated Carbon for Aiding in Cleaning Basins.—At an industrial plant it was found that settlings from coagulated water could be removed much more easily when powdered activated carbon was used in the coagulated water.

Calibrated Velocity Head Rod for Measuring Stream Flow.—Two engineers of the Forest and Range Experiment Station found that a calibrated rod placed in a stream could be used to measure velocity head by the height of water against the rod and flows thus determined with good accuracy. [Committee Note—One of the members has used this method occasionally in years past.]

Radio Channel Allocations for the Public Water Supply Industry.—In September, 1943, a "Radio Technical Planning Board" was organized by several associations within the Radio industry. A committee of the American Water Works Association worked in conjunction with this Board, assembled data on the radio needs of water supplies, and participated in a hearing in September, 1944, before the Federal Communications Commission. The Commission's findings are not known to the Committee on Water Supply Engineering.

Respectfully submitted,

F. A. BARBOUR
J. R. BAYLIS
W. W. BRUSH
J. S. LONGWELL
GEORGE D. NORCOM
THOMAS H. WIGGIN, *Chairman*

Committee of the Sanitary Engineering Division on Water Supply Engineering

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DISCUSSIONS

COEFFICIENTS FOR VELOCITY DISTRIBUTION IN OPEN-CHANNEL FLOW

Discussion

BY WILLIAM S. EISENLOHR, JR.

WILLIAM S. EISENLOHR, JR.,[15] ASSOC. M. AM. SOC. C. E.[15a]—The thesis of this paper, as given in the "Synopsis" and "Introduction," is that there are problems in nonuniform flow in which an appreciable part of the total change in momentum and kinetic energy between two cross sections may be due to a change in the velocity distribution as distinguished from a change in the mean velocity. The Chézy and Manning equations are momentum equations. Thus, the coefficient of velocity distribution should be the ratio of the mean of the squares of the local velocities to the square of the mean velocity. That the several discussions of the paper, although critical, have not successfully disproved this thesis will be shown. To do so, however, it will be necessary to present revised demonstrations. At the same time, an effort will be made to clarify some of the original statements.

First, it is necessary to include the concept embodied in Eq. 27 given by Professor Van Driest and derived from the paper by B. A. Bakhmeteff, M. Am. Soc. C. E. Somehow or other, in preparing the paper, the writer lost sight of the concept that there is an energy equation involving the square relation as well as one involving the cube relation of the velocity. Since the difference between momentum and energy no longer is sufficient to account for the use of different coefficients, it becomes appropriate to redesignate the coefficients as follows: C_s (formerly C_m) $= \dfrac{\overline{u^2}}{\overline{V^2}}$ = coefficient of velocity distribution based on the second power of the velocity for use in space equations; and C_t (formerly C_e) $= \dfrac{\overline{u^3}}{\overline{V^3}}$ = coefficient of velocity distribution based on the third power of the velocity for use in time equations. (The writer prefers the use of the letter C with subscripts, to that of the Greek letter α, since the

NOTE.—This paper by William S. Eisenlohr, Jr., was published in January, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1944, by A. A. Kalinske, and Edward H. Taylor; and September, 1944, by E. R. Van Driest, and Hunter Rouse and John S. McNown.

[15] Associate Hydr. Engr., Water Resources Branch, U. S. Geological Survey, Washington, D. C.

[15a] Received by the Secretary March 15, 1945.

latter cannot be made on an ordinary typewriter and is difficult to make attractively by hand.)

The writer stated that the condition of steady, gradually varied flow would be assumed. Perhaps this point should have been given more emphasis. By steady flow is meant that there is no variation in the conditions of flow with respect to time. Gradually varied flow is a condition of flow in which the convergence, divergence, or curvature of the stream tubes occurs so gradually that consideration of the stream tubes as parallel will introduce no appreciable error. In other words, the assumption is made that the vector velocity v can be replaced by u without appreciable error. (A similar assumption is always made in computing the slope S, the quantity $\tan\theta$ being used for $\sin\theta$.) Having made this assumption, the careful distinction between the two made by Messrs. Rouse and McNown is without significance and somewhat confusing. This assumption also gives the element of direction which Professor Taylor finds lacking in Eq. 2.

With the foregoing assumptions always in mind, the equations for gradually varied flow in open channels will be developed. The basic equation is the momentum equation expressing Newton's second law of motion—the rate of change of momentum is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts. Therefore:

$$dF_R = dm\frac{d\mathrm{v}}{dt} \qquad (40)$$

in which F_R is the resultant force and v is the vector velocity. By the assumption of gradually varied flow, the velocity component parallel to the x-axis, u, can be substituted for v without appreciable error. Thus, the resultant force must be parallel to the x-axis and must be equal to the sum of the components in the x-direction of all forces acting, ΣdF_x. Then, $\Sigma dF_x = dm\frac{du}{dt}$ will describe the change in momentum completely. Substituting $\frac{du}{dt} = \frac{du}{dL}\frac{dL}{dt} = u\frac{du}{dL} = \frac{1}{2}\frac{du^2}{dL}$, the momentum equation is

$$\Sigma dF_x = dm\left(\frac{1}{2}\frac{du^2}{dL}\right) \qquad (41a)$$

Eq. 41a is a momentum equation but the rate of change of momentum is expressed in terms of distance L rather than in terms of time t. As stated by Messrs. Rouse and McNown, u is a function of L, and Eq. 41a cannot be integrated rigorously without taking that relation into consideration. However, it will be shown subsequently that, if the acceleration is assumed constant (or the equivalent of using the average force $\frac{1}{L}\int_0^L \Sigma dF_x\, dL$ as stated by Professor Taylor), reasonably close results for gradually varied flow will be obtained. Eq. 41a is readily transformed into the energy equation—

$$\Sigma dF_x\, dL = \tfrac{1}{2}\, dm\, du^2 \qquad (41b)$$

—the energy equation in which the work done by the force dF_x equals the change in kinetic energy of the mass dm.

Whether Eq. 41*a* is transformed into Eq. 41*b* by transposing dL or by multiplying both sides by dL, the result is the same—a vector force is multiplied by the distance through which it acts to give the scalar quantity of work. Likewise, the right side of Eq. 41*a* has the dimensions of the rate of change of momentum, $\frac{M\,L^2}{T^2\,L} = \frac{M\,L}{T^2}$, which is a vector quantity. Whether the quantity is multiplied by L, or the divisor L is removed, the result is $\frac{M\,L^2}{T^2}$, the dimension of energy, which is a scalar quantity. There is nothing novel in such a procedure. In fact, it is the standard derivation of the work-energy equation. This transformation is not rigorous except in the differential form.

If both sides of Eq. 41*b* are divided by dt: $\Sigma dF_x \frac{dL}{dt} = \frac{1}{2}\,dm\,\frac{du^2}{dt}$ or $\Sigma dF_x\,u = \frac{1}{2}\,dm\,[du^2]_{t=1}$, the latter being the differential form of Eq. 3*b*. Without imposing the restriction of $t = 1$, and making the substitution $\frac{du}{dt} = u\,\frac{du}{dL}$, the power equation is obtained:

$$\Sigma dF_x\,u = dm\left(\frac{1}{3}\frac{du^3}{dL}\right) \qquad (41c)$$

Eq. 41*c* is frequently called an energy equation (as was Eq. 3*b* in the paper); but, since it actually represents changes in work and energy per unit time, it will be designated the power equation—especially to distinguish it from Eq. 41*b*.

From the three basic equations of momentum, energy, and power, the three equations of nonuniform flow in an open channel will be derived. Considering Eqs. 41*a*, 41*b*, and 41*c*, the coefficient of velocity distribution seems to be based on the square relation in the momentum and energy equations and on the cube relation in the power equation. This theory will be shown to be correct.

The simplest way to obtain the equations for the entire channel would be to let $dm = \rho\,dA\,dL$ and then to perform the double integration with respect to A and L. To avoid calculus, where possible, and at the same time to give emphasis to the physical significance of the equations, the mass will be taken as the entire mass of the stream tube $\rho\,L\,\overline{dA}$, in which L is the length and $\overline{dA}$ is the average value of the cross section. Thus, $L\,\overline{dA}$ equals the volume of the stream tube. Such a stream tube is shown in Fig. 5, the boundaries of which are fixed in space. In the paper it was found that selecting a volume of water and following the change in energy in that volume with respect to time did not yield practical results; therefore, such a method will not be given further consideration.

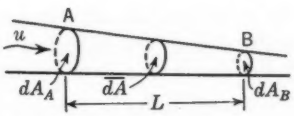

FIG. 5.—STREAM TUBE (VOLUME = $L\,\overline{dA}$)

Although the stream tube is fixed in space, the water particles at any point within the tube will change from instant to instant. In steady flow, each particle arriving at a given point will have exactly the same momentum or energy possessed by the particle which just left that point. As a result, the total momentum or energy contained within the stream tube will be the same at every instant of time. During any interval of time a volume of water dB will flow out of the stream tube, and by the continuity of flow an equal volume dB will flow into the stream tube. Thus, the total change in momentum or energy of the water flowing through the stream tube during a given interval will be represented by the difference between the momentum or energy of a volume dB leaving the tube and the momentum or energy of a similar volume entering the tube. The three equations of flow can be developed as follows.

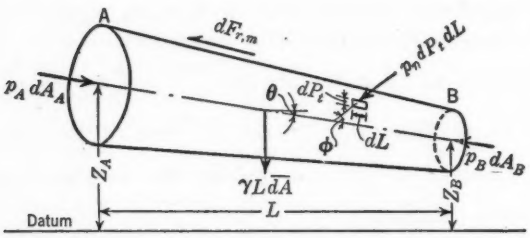

FIG. 6.—FORCES ACTING ON STREAM TUBE

Revised Development of the Momentum Equation.—In this revised demonstration the effect of the variation of dA with L will be fully considered. The rate of change of momentum is equal to the resultant of all the external forces that are acting. For the stream tube in Fig. 6, the pressure forces are, as before:

$$\int_A^B dF_p = p_A\,dA_A - p_B\,dA_B - \bar{p}_n\,(dA_A - dA_B) \ldots\ldots\ldots (42a)$$

in which the first two terms on the right side represent the pressure forces on the end areas and the third term represents the component in the direction of motion of the normal pressure p_n on the sloping sides of the tube. Since this force is important and usually neglected, it will be considered in detail.

The pressure on any element of area along the side of the stream tube is $p_n\,dP_t\,dL$ in which P_t is the perimeter of the stream tube. For uniform flow, p_n is normal to the direction of flow and the term disappears. In gradually varied flow, p_n is still the normal pressure on the side of the stream tube; but, by reason of the change in area, the side must make a slight angle and p_n an angle ϕ with the direction of flow. The pressure force on the side of the stream tube now has a component in the direction of flow equal to $p_n\,dP_t\,dL\cos\phi$. The total force is then $\int\int p_n\,dP_t\,dL\cos\phi$; but $\int\int dP_t\,dL\cos\phi$ equals the difference in end areas of the stream tube. The value of $\bar{p}_n$ is defined by the relation: $\bar{p}_n\,(dA_A - dA_B) = \int\int p_n\,dP_t\,dL\cos\phi$.

As it is impracticable to attempt to evaluate $\bar{p}_n$, Eq. 42*a* will be written in a more usable form by expressing the forces in terms of the average area $\overline{dA}$. The quantity $\overline{dA}$ has such a value that the total volume of the stream tube between sections A and B is $L\,\overline{dA}$. Let k_1 be a certain portion of the difference

in end areas, having a value such that $dA_A = \overline{dA} + k_1 (dA_A - dA_B)$ and $dA_B = \overline{dA} - (1 - k_1) (dA_A - dA_B)$. Likewise, let $p_n = p_A - k_2 (p_A - p_B)$. Substituting these values in Eq. 42*a* gives, for the pressure forces:

$$\int_A^B dF_p = (p_A - p_B)\, \overline{dA} + (k_1 + k_2 - 1)\, (p_A - p_B)\, (dA_A - dA_B) \ldots (42b)$$

The gravity force $\int_A^B dF_g$ is $\gamma\, L\, \overline{dA} \sin\theta$, and the resisting force is $dF_{r,m}$. The change in momentum during any interval, as explained previously, can be represented by the difference between the momentum of the volume of water dB as it leaves the stream tube at section B and the momentum of an equal volume of water dB as it enters the stream tube at section A. This change can be expressed as $\rho\, dB\, u_B - \rho\, dB\, u_A$. Since the resultant force is equal to the rate of change of momentum, the volume dB in this case must be the volume change per unit time, dQ. Therefore,

$$\int_A^B \Sigma dF_x = \rho\, dQ\, (u_B - u_A) = \rho\, u^2{}_B\, dA_B - \rho\, u^2{}_A\, dA_A \ldots\ldots (42c)$$

Eq. 42*c* is identical with the expression derived by Messrs. Rouse and McNown through their integration process. In Eq. 42*c*, there was no need for integration because the total change in momentum for the entire length of stream tube L was used. Since the pressure forces are not amenable to practical application when expressed in terms of dA_A and dA_B, Eq. 42*c* will be written in terms of the average area $\overline{dA}$. Let $dQ = u'\, \overline{dA}$ and $u' = u_A + k_3\, (u_B - u_A)$ (in which k_3 is some portion of the difference in velocity between the ends of the stream tube) and substitute in Eq. 42*c*:

$$\int_A^B \Sigma dF_x = \rho\, \overline{dA}\, [k_3\, u^2{}_B - (1 - k_3)\, u^2{}_A + (1 - 2\, k_3)\, u_A\, u_B] \ldots (42d)$$

Combining the various parts, the general momentum equation is obtained for gradually varied flow in a stream tube between sections A and B in terms of the average area of the tube $\overline{dA}$:

$$(p_A - p_B)\, \overline{dA} + (k_1 + k_2 - 1)\, (p_A - p_B)\, (dA_A - dA_B) + \gamma\, L\, \overline{dA} \sin\theta - dF_{r,m} = \rho\, \overline{dA}\, [k_3\, u^2{}_B - (1 - k_3)\, u^2{}_A + (1 - 2\, k_3)\, u_A\, u_B] \ldots (42e)$$

Eq. 42*e* is complicated by the proportionality factors k_1, k_2, and k_3. These factors represent the portion of the difference between the values at sections A and B by which the average value of the area, pressure, and velocity is greater or less than the value at section A. It seems reasonable to suppose that for gradually varied flow the values of k_1, k_2, and k_3 will be so close to 0.5 that they can be assumed to have that value. Making this assumption, Eq. 42*e* becomes, after substituting $\sin\theta = \frac{Z_A - Z_B}{L}$ (actually $\tan\theta$) and

rearranging:

$$dF_{r,m} = \gamma \left[\left(\frac{p_A}{\gamma} + Z_A \right) \overline{dA} - \left(\frac{p_B}{\gamma} + Z_B \right) \overline{dA} + \frac{u^2_A}{2g} \overline{dA} - \frac{u^2_B}{2g} \overline{dA} \right] \ldots (43)$$

which is identical with Eq. 6 except that the area of the stream tube is definitely labeled the average area. The only assumption is that the average area, pressure, and velocity are the averages of the values at the end sections. The term $\left(\frac{u^2_A}{2g} - \frac{u^2_B}{2g} \right) \overline{dA}$ is the practical equivalent of $\frac{u^2_A}{g} dA_A - \frac{u^2_B}{g} dA_B$. The same answer would be obtained if d'Alembert's principle of reversed effective force were used, and the momentum changes were represented by pressure forces equal to the velocity heads. These pressures on the end areas, of course, are $\left(\frac{u^2_A}{2g} \right) dA_A$ and $\left(\frac{u^2_B}{2g} \right) dA_B$; but the average pressure on the difference in end area must also be considered and, when it is included, the result is the same as Eq. 43.

The integration of Eq. 43 with respect to A yields Eq. 7 as before, and Eqs. 9 to 12 remain unchanged. The symbol $F_{r,m}$ denotes the total boundary shear and is usually assumed to be the product of the average unit shear and the area of the boundary. Professor Kalinske has stated that such an assumption limits the length of channel over which the expression is valid. On the other hand, the length of channel must be long enough that the effect of boundary shear can be measured with reasonable accuracy. Although these conditions limit the application of Eq. 9*a*, the principle involved is the momentum theory of external forces that are acting, and the substitutions made in arriving at the Chézy and Manning equations are empirical expressions for boundary shear. As Professor Rouse has stated[16] for the Manning equation, "The factor n is evidently a measure of boundary roughness, since it is invariably tabulated solely as a function of boundary composition." Furthermore, Robert Manning's studies leading to the development of the "Manning" equation included results of experiments on the Hayne (Belgium), Parana (South America), and Mississippi rivers; and modern tables of values of n almost always include values for natural streams. The statement by Messrs. Rouse and McNown that "Actually, the [familiar resistance] equations in question have been derived solely for the case of uniform flow * * *" thus appears to be unduly restrictive.

Revised Development of the Energy Equation (See Heading, "Development of the Power Equation" for Revised Development of the "Energy Equation" of the Paper).—The change in kinetic energy represented by the difference between the energy of volume dB, as it leaves the tube at section B, and the energy of an equal volume dB, as it enters the stream tube at section A, is the result of the work done by all forces acting on the tube during the same interval of time. Since work is a scalar quantity, the total work done is the algebraic sum of the product of force times distance moved in the direction of the force. The

[16] "Evaluation of Boundary Roughness," by Hunter Rouse, *Proceedings*, 2d Hydr. Conference, June 1–4, 1942, State Univ. of Iowa, Iowa City, *Bulletin No. 27*, Studies in Eng., 1943, p. 108.

work done by the pressure forces is the sum of the pressure force on each bounding surface of the tube times the distance of movement across that surface in the interval of time considered. Since there is no movement across the sides of the tube, the only work done by the pressure forces is on the ends of the tube. To visualize this work, assume the stream tube to be separated in space, as in Fig. 7. The water will approach the tube as a filament of uniform cross section, dA_A, and will leave with a uniform cross section, dA_B. If the volume passing through the tube in a given time interval is defined as $dB = s_A\,dA_A = s_B\,dA_B$ (in which s is a small distance in the direction of flow, the dL of Eq. 41*b*), the total work done by the pressure forces is $p_A\,dA_A\,s_A = p_B\,dA_B\,s_B$.

FIG. 7.—STREAM TUBE AS A FREE BODY SHOWING WORK DONE BY PRESSURE FORCES ($s_A\,dA_A = s_B\,dA_B$)

The total work done by the gravity force is equivalent to the weight of volume dB times the distance volume dB moves in the direction of the gravity force while being moved from section A to section B. This distance is the difference in elevation, $Z_A - Z_B$. The work done by gravity is therefore $\gamma\,dB\,(Z_A - Z_B)$. The change in kinetic energy, of course, is $\rho\,dB\,\frac{u^2_B}{2} - \rho\,dB\,\frac{u^2_A}{2}$.

During the interval required for volume dB to enter or leave the stream tube, work is done by the water in the tube against the shearing force at the sides. There is also a dissipation of energy as heat from the turbulence set up by the resistance of the sides of the tube. The net result of these effects is a loss of energy within the tube that, for convenience, will be expressed as a loss of potential energy $\gamma\,dB\,h_L$, in which h_L is the energy lost per unit weight of water passing through the stream tube, usually referred to as the loss of head. The work-energy equation is, then:

$$p_A\,dA_A\,s_A - p_B\,dA_B\,s_B + \gamma\,dB\,(Z_A - Z_B) - \gamma\,dB\,h_L = \rho\,dB\,\frac{u^2_B}{2} - \rho\,dB\,\frac{u^2_A}{2} \quad \ldots\ldots (44a)$$

Rearranging, and substituting $dB = s_A\,dA_A = s_B\,dA_B$, gives

$$(p_A + \gamma\,Z_A)\,s_A\,dA_A - (p_B + \gamma\,Z_B)\,s_B\,dA_B. = \rho\,\frac{s_B}{2}\,u^2_B\,dA_B - \rho\,\frac{s_A}{2}\,u^2_A\,dA_A + \gamma\,h_L\,dB \quad \ldots\ldots (44b)$$

It must be remembered that in Eqs. 44 the change in energy of the volume dB during the displacement from A to B was used as the equivalent of the change in energy of the stream tube during the displacement s. In integrating over the cross section, therefore, the volume B obtained is that resulting from the displacement s of the mass of the entire stream between A and B (see Eq. 41*b* and subsequent discussion). Hence s_A and s_B are both constant.

Proceeding with the integration, making use of the coefficient C_s previously defined, and letting $h_s = \frac{1}{B}\int_A h_L\, dB$ (in which h_s is the average for the entire stream of the energy lost per unit weight because the stream passed over the length of channel L), the following expression results, after dividing by the weight of the volume B:

$$\frac{p_A}{\gamma} + Z_A + C_{s,A}\frac{V^2_A}{2g} = \frac{p_B}{\gamma} + Z_B + C_{s,B}\frac{V^2_B}{2g} + h_s \ldots\ldots (44c)$$

Development of the Power Equation.—Power is the time rate of doing work. To obtain the power equation it is only necessary to divide the work-energy formula, Eq. 44*b*, by the time interval Δt during which the volume dB entered or left the stream tube. Since s is a small distance, $\frac{s}{\Delta t} = u$. Performing the division:

$$(p_A + \gamma Z_A)\, u_A\, dA_A - (p_B + \gamma Z_B)\, u_B\, dA_B = \frac{\rho}{2} u^3_B\, dA_B - \frac{\rho}{2} u^3_A\, dA_A + \gamma h_L\, u\, dA \ldots\ldots (45a)$$

Integrating Eq. 45*a* with respect to A by making use of the coefficient C_t previously defined, letting $h_t = \frac{1}{V A}\int h_L\, u\, dA$ (in which h_t is the average for the entire stream of the energy lost per unit weight per unit time because the stream passed over the length of channel L), and then dividing by the weight per unit time, γQ:

$$\frac{p A}{\gamma} + Z_A + C_{t,A}\frac{V^2_A}{2g} = \frac{p_B}{\gamma} + Z_B + C_{t,B}\frac{V^2_B}{2g} + h_t \ldots\ldots (45b)$$

Comparison of Equations.—Three equations have been developed for gradually varied flow in open channels as follows:
Momentum—

$$\left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2g}\right]_A \approx \left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2g}\right]_B + \frac{\bar{\tau}_0 \bar{P} L}{\gamma \bar{A}} \ldots\ldots (46a)$$

energy—

$$\left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2g}\right]_A = \left[\frac{p}{\gamma} + Z + C_s\frac{V^2}{2g}\right]_B + h_s \ldots\ldots (46b)$$

and power—

$$\left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}\right]_A = \left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}\right]_B + h_t \ldots\ldots (46c)$$

The approximation symbol is used with the momentum formula (Eq. 46*a*) to emphasize the fact that the equality exists only under certain specified conditions and to aid in the comparison with the energy formula, Eq. 46*b*. In the following discussion it will be assumed that the conditions of gradually varied flow are such that the approximation becomes an equality.

Eq. 46*a* indicates that the momentum equation can be put in the form of the Bernoulli equation only by making restrictive assumptions. Therefore, the statement made in the "Summary" of the paper that "* * * the Bernoulli equation is also a momentum equation * * *" appears to be erroneous if the Bernoulli equation of ordinary hydraulics is considered, despite the fact that at least seven modern textbooks on hydraulics and fluid mechanics give the derivation based on the rate of change of momentum as one proof of the Bernoulli equation. Examination of these textbooks discloses that in only one is any consideration given to a variation in cross-sectional area and that one[17] disposes of the variation in a manner somewhat similar to the writer's original development. In uniform flow there is no variation in area and consequently no change in velocity; thus, the velocity terms cancel out of the equation. Obviously, the Bernoulli equation without the velocity terms is not the Bernoulli equation. Furthermore, in applying the momentum theorem to local phenomena (the hydraulic jump, for example), approximations cannot be made that are valid for gradually varied flow, and the similarity to the Bernoulli equation becomes more remote. Therefore, it is not proper to derive the Bernoulli equation of ordinary hydraulics from the momentum theorem.

The similarity of equations derived by the momentum and energy principles has been emphasized to show how readily the two can be confused. Thus, by rearranging Eqs. 46*a* and 46*b*, the following expressions are obtained:

$$\frac{\bar{\tau}_0 \bar{P} L}{\gamma \bar{A}} \approx \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2g}\right]_A - \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2g}\right]_B \ldots\ldots (47a)$$

and

$$h_s = \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2g}\right]_A - \left[\frac{p}{\gamma} + Z + C_s \frac{V^2}{2g}\right]_B \ldots\ldots\ldots (47b)$$

Each term in Eqs. 47 has the dimension of length. Therefore, as lengths, the right sides of the two equations are identical. However, these lengths represent different physical quantities. The lengths in Eq. 47*b* represent the usual energy per unit weight because they were obtained by dividing the energy equation by γB, the weight of water passing through the reach L. On the other hand, the lengths in Eq. 47*a* represent force per unit area because they were obtained by dividing the momentum equation by γA. The significance of the equality of the right sides of the two equations is that, for conditions satisfied by Eq. 47*a*, the loss of energy within the stream is the result of boundary shear alone.

The principal distinction between Eqs. 47*a* and 47*b* is that the latter is rigorous because it is the definition of the quantity h_s and is not necessarily limited to gradually varied flow, whereas the former defines the resistance to flow as the boundary shear for the specified conditions of flow. Since the length given by the right side of Eq. 47*a* is defined as h_s in Eq. 47*b*, it is permissible to write $\frac{\bar{\tau}_0 \bar{P} L}{\gamma \bar{A}} \approx h_s$ as long as h_s is considered as a length only. If

[17] "Hydraulics and Its Applications," by A. H. Gibson, 4th Ed., Constable & Co., Ltd., London, 1934, pp. 82–84.

h_s is considered as loss of energy per unit weight, the foregoing substitution is not permissible because it implies that h_s is to be multiplied by γB and this would make the expression nonhomogeneous, since there is also the implication that the left side is to be multiplied by γA. In other words, making approximations that involve little error and writing the momentum equation in the form of Eq. 47*a* (the right side of which is identical with the energy formula, Eq. 47*b*) do not make Eq. 47*a* an energy equation. Such a procedure, however, is apparently what Messrs. Rouse and McNown visualize as a basis for the statement,

> "The customary loss coefficients of nonuniform flow * * * are simply means of evaluating the quantity H_l of Eqs. 37 and 39 in terms of the boundary geometry and the Reynolds number, and hence have no source whatever in the momentum principle."

The coefficient of velocity distribution is C_s, the ratio of the mean of the squares of the local velocities to the square of the mean velocity. Further examination shows that both the resisting force and the energy lost are related to the length of channel L without consideration of the time factor. The coefficient C_s, therefore, is the proper coefficient for use in space equations.

Turning to the relation between the energy and power equations, it will perhaps be simpler to show the difference by using an illustration. Consider a stream flowing out of a large lake. At cross section A near the lake, the velocity distribution will be very uniform and no appreciable error will be involved by assuming $C_s = C_t = 1.00$. This particular condition was chosen to simplify the demonstration. Suppose that at a second cross section B some distance downstream, away from entrance conditions, the stream channel is complex and the value of C_s is 1.2 and that of C_t is 1.6 (which is considered reasonable for a complex natural channel). Conditions will then be as shown in Fig. 8.

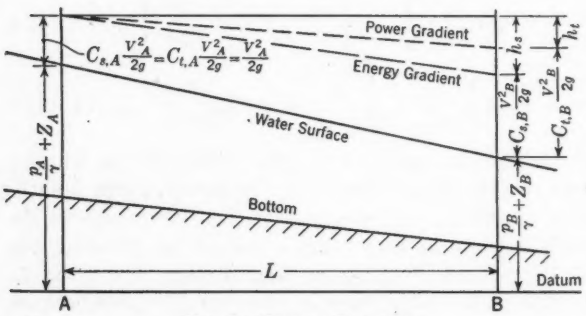

FIG. 8.—PROFILE DIAGRAM

In the development of the energy equation, h_s was defined as the average energy lost per unit weight in the distance L between sections A and B. Therefore, from Eq. 47*b* it appears that the quantity $\frac{p}{\gamma} + Z + C_s \frac{V^2}{2g}$ must be the energy possessed by a unit weight of water passing any section. Since $\frac{p}{\gamma} + Z$ represents the elevation of the water surface, the point plotted a distance $C_s \frac{V^2}{2g}$ above the water surface in a profile diagram such as Fig. 8 must represent

the energy content per unit weight passing that section. A line connecting all such points would then be the "energy content line." Obviously, for steady flow the energy content line will not change with time and its slope becomes the "energy gradient" when this term is defined as the rate of loss of energy per unit weight per unit length of channel. The distinction between "energy content line" and "energy gradient" is important only in the case of unsteady flow.

In the same way the power equation can be written in the form (compare Eq. 47*b*):

$$h_t = \left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}\right]_A - \left[\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}\right]_B \quad \ldots\ldots\ldots (48)$$

and the quantity $\frac{p}{\gamma} + Z + C_t\frac{V^2}{2g}$ is the average energy per unit weight passing any section per unit time, or the power at any section. Likewise, in a profile diagram such as Fig. 8, a line plotted a distance $C_t\frac{V^2}{2g}$ above the water surface will be the "power content line" which for steady flow has a slope that represents the average loss of energy per unit weight per unit time per unit length of channel and is the "power gradient."

Thus, h_s is the average energy lost by the stream for every pound of water passing through the reach from section A to section B, but h_t is the average energy lost per second per pound of water passing through the reach. Although h_s and h_t have different numerical values, they represent the same physical quantity—energy lost by the stream in distance L. Therefore, it should be possible to obtain h_s by multiplying h_t by $\bar{t}$, the average time needed for a pound of water to pass any section. To do this, since both h_L and t vary for each stream tube, the multiplication must be made before integration as in the development of the base equations.

The coefficient of velocity distribution for use in the power equation (where the loss of energy per unit time is considered) is C_t, the ratio of the mean of the cubes of the local velocities to the cube of the mean velocity.

The use of the energy or power equation is largely a matter of choice. In either case, coefficients for loss of energy will be determined experimentally by using the equation selected. There are certain considerations that affect the choice of equation. Since the total loss of energy depends on the volume passing any cross section, the most convenient volume to use is Q, the volume per unit time. This leads to the selection of the power equation and (since power is energy per unit time) the frequent designation of the power equation as the energy equation.

There may be two objections to such a procedure. In the first place, even though the energy equation or the power equation is the logical choice for the solution of a given problem, the effect of friction will usually be obtained by the use of the Manning equation or the Chézy equation. Since these are momentum equations and only the energy equation uses the same velocity distribution coefficient, C_s, the energy equation may be more suitable for general use than the power equation. Secondly, the velocity distribution

coefficients are usually ignored. Disregarding them in the power equation will introduce about three times the error caused by disregarding them in the energy equation. To avoid confusion it is suggested that the name "Bernoulli equation" be used only in connection with the flow through a single stream tube or through a channel with uniform velocity distribution.

Several discussers have suggested that since the velocity distribution coefficients were small, especially in relation to other unknowns, one should neglect them rather than become concerned over their proper use. For those accustomed to thinking of open channels in terms of canals or similar channels of regular cross section, such a procedure may seem appropriate. The writer believes, however, that the velocity-distribution coefficient will become an important tool in the analysis of flow in natural channels of complex section. There are indications that values of C_s and C_t for such channels can easily be as great as 1.2 and 1.6, respectively, and can vary quite rapidly from section to section. This paper has been presented for the benefit of those engineers who find themselves confronted with situations in which velocity-distribution coefficients must be considered. In closing, the writer desires to thank all those who offered discussions. The critical comments led the writer to make such extensive revision of the paper. He hopes the result will be found sufficiently rigorous and suitable for explaining some of the confusion on the subject.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOOD FORMULAS BASED ON DRAINAGE BASIN CHARACTERISTICS

Discussion

BY H. B. KINNISON AND B. R. COLBY

H. B. KINNISON,[42] M. AM. SOC. C. E., AND B. R. COLBY,[43] ESQ.[43a]—The excellent discussions of this paper are deeply appreciated, and they strengthen the writers' belief that the study of the frequency and magnitude of floods should have the best efforts of many hydraulic engineers. Although records of stream flow are much more numerous than they were only a few years ago, they still are inadequate for most types of investigations; and, therefore, differences of opinion and divergent results are to be expected. The writers hope that the paper and the discussions are a constructive contribution to the field of flood studies.

The discussion by Mr. Jarvis presents a comparison between the Modified Myers formula and the writers' formulas, and states that "No matter how involved any preferred procedures, the results are readily translated into terms of Myers percentage ratings, as demonstrated by Table 2." The form of the equation, $Q = C_m \sqrt{M}$, makes it possible to convert the results of any procedure which arrives at a value of Q into a percentage rating, as the percentage rating becomes the only unknown. However, the very fact that the percentage ratings in Table 2 show so wide a variation for a flood of a given magnitude is an indication of the limitations of this type of formula. For instance, for "rare floods," the percentage rating varies from 5.4 to 51.0; and, in relation to the highest observed flood, the "rare flood" varies from 81% to more than 300% for those streams having more than 10 years of record. The engineer faced with the problem of choosing a reasonable percentage rating for a particular stream must seek another procedure rather than the Modified Myers formula to properly evaluate the flood-producing characteristics of that stream.

NOTE.—This paper by H. B. Kinnison, M. Am. Soc. C. E., and B. R. Colby, Esq., was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Clarence S. Jarvis; September, 1944, by L. R. Beard, and M. J. Ord; October, 1944, by Howard M. Turner, and Byron O. McCoy; and November, 1944, by C. O. Clark.

[42] Dist. Engr., U. S. Dept. of Interior, Geological Survey, Boston, Mass.

[43] Associate Engr., U. S. Dept. of Interior, Geological Survey, Iowa City, Iowa.

[43a] Received by the Secretary April 7, 1945.

Unit hydrographs of streams with the same drainage area show wide divergence in peak discharge, even though the basins may be adjacent to each other. The writers' formulas were developed in an attempt to evaluate the effect of those physical characteristics which cause so much divergence in the flood peaks.

In Table 2, Mr. Jarvis has given the computed peaks of the minor, major, rare, and maximum floods in percentage ratings based on the Modified Myers formula. He concluded from this table that the maximum flood peaks are too high. Mr. Clark states that they are probably too low. The maximum flood peak as defined in the paper has an infinite recurrence interval, or a probability of occurrence approaching zero. It is a computed peak representing the upper limit of possible peaks. Because ratings computed from the Modified Myers formula have been derived almost entirely from observed flood peaks or for peaks of probable or design floods, ratings computed for maximum peaks will naturally seem to be very high. The maximum peaks in Table 2 that have percentage ratings greater than 90% are not inconsistent with the other maximum peaks. They are all of the same order of magnitude in that they have the same periodicity. The largest percentage for a maximum peak (right-hand half of Col. 12, Table 2) is only 8.1 times the smallest percentage for a maximum peak; but the largest percentage for a minor peak (right-hand half of Col. 9, Table 2) is 13.2 times the smallest percentage for a minor peak.

If Mr. Jarvis means that the maximum probable flood peak should be about 150% of the rare flood peak, the writers agree that this would be a reasonable assumption. As stated in the paragraph preceding the heading, "Physical Characteristics," the peak discharge of the maximum probable summer flood might be "about 45% of the peak discharge of the maximum flood."

Mr. Jarvis found from his analysis that the formulas should not be applied to basins having characteristics that differ widely in any important physical characteristic from the basins used in the study. This limitation agrees with the stated conclusions of the writers. They are indebted to Mr. Jarvis for stressing this fact.

In his criticism of the factors included in Eq. 6, Mr. Jarvis has misinterpreted the effect of different physical characteristics on computed flood peaks, and his criticism of the use of L and a in Eq. 6 is not valid. The effect of the characteristic L on computed flood peaks must be considered in relation to the changes in other characteristics which are normally associated with changes in L. Similarly, any variation in the pondage factor a is invariably associated with changes in the slope factor s, and usually with changes in L, at least in central New England where the formulas are considered applicable.

Entirely different formulas might have been developed on the basis of Mr. Jarvis' assumptions. However, the writers preferred to develop formulas from relationships which were indicated by the basic data, and which fit the data for this particular locality closely.

The original report of the study on which the paper was based (see "Acknowledgments") was prepared for use in Massachusetts and possibly in some

adjoining areas. When the paper was prepared for wider circulation, some changes might have been made. As Mr. Ord suggests, the formulas, from the original report, might well have been shown by graphs. Also, a discussion of the floods of March, 1936, and September, 1938, should have been included, so that any one not familiar with these floods on Massachusetts streams would not have questioned the fact that these were greater floods than would normally occur during the short period covered by gaging station records.

Mr. McCoy's observation that the exponents and coefficients in the formulas should be consistent for different classes of floods is a logical one. In determining the effects of the various characteristics, each of the four classes of floods was considered separately, and correlations indicated by the basic data were used without adjustment. The writers felt that, because of the limited knowledge of the influence of each characteristic, complete dependence should be placed on the basic data.

Mr. Beard and others question the accuracy of curves C and D in Fig. 7. These curves are not well defined, but they are based on logical extensions of the available data. Discussion of the derivation of these curves was considered much too lengthy to be given in the paper. However, it should be understood that runoff coefficients were not confined to a narrow range. For example, precipitation in the range from 5.6 in. to 6.0 in. (the range was expressed to the nearest tenth of an inch) was found to have about one chance in seven of generating less than 1.0 in. of runoff. Under extreme conditions of precipitation falling on snow cover, precipitation was assumed to have less than one chance in one thousand of generating more than 10 in. of runoff.

Depth of runoff was not determined solely from curve D which was based directly on incompletely defined relations between rainfall and runoff. The basis for the depths of runoff that were finally used is stated in the paragraph preceding the heading, "Maximum Flood Runoff."

In Table 3, Mr. Ord lists the minor, rare, and maximum flood peaks in percentages of the major flood peaks. Fig. 17, showing a curve drawn through the average percentages for each class of flood peak, is interesting and could be used to compute flood peaks having other recurrence intervals after the major flood peak had been computed. However, the writers do not agree that flood peaks computed from this type of average curve would be as accurate as flood peaks computed from the frequency curves for individual drainage areas. Although most frequency curves for basins in the same region probably should be approximately parallel when drawn on logarithmic paper, each one is not necessarily parallel to all the others. Also, the use of an average curve seems to be inconsistent with the suggestion made by Mr. Ord that the floods of March, 1936, and of September, 1938, should have been given more weight in the study. Figs. 1 and 11 show how difficult it would be to draw parallel curves for the three basins if much emphasis were given to the floods of March, 1936, and of September, 1938.

Mr. Ord states, "The percentages for the maximum flood have a greater variation." He then adopts the doubtful assumption that those maximum peaks which show the extreme variations from the average should be questioned. The maximum flood peaks have a greater variation only because the maximum

flood peaks are farthest from the major flood peaks, which Mr. Ord makes coincide. If the maximum flood peaks were made to coincide, the peaks of the minor floods would have had the greatest variation. Table 2 shows that the ratio of the highest to the lowest maximum flood peak is less than the corresponding ratio for any other class of flood peak.

Mr. Clark and Mr. Ord prefer to make the unit hydrograph a direct basis for computations of expected flood peaks. That is a logical procedure, and the study reported in the paper was begun on a similar plan, but that plan was soon abandoned. The computed unit hydrograph is an imperfect foundation on which to build. As Mr. Clark states, the unit hydrographs obtained by investigators using different methods may not agree. For flashy streams, the peaks of the unit hydrographs computed by the same person from several flood hydrographs may differ widely. Also (on Massachusetts streams at least), the peaks of computed unit hydrographs vary with the depth of runoff. Publication of depths of runoff to be used with unit hydrographs computed by other engineers might lead to very inaccurate computations of flood peaks. Besides convenience of application, the correlation of computed peaks for each class of flood with the physical characteristics of drainage areas has the advantage of insuring at least a consistent computation and the use of unit hydrographs. Also, the peaks of floods computed from unit hydrographs having sharp peaks are not as variable as the peaks of the unit hydrographs.

The writers were reluctant, as is Mr. Turner, to assign numerical recurrence intervals to the different classes of floods because of the accuracy which numerical intervals connote. However, adjective ratings alone are indefinite and would have a wide range of interpretation. Also, some assumption of recurrence intervals was needed for plotting. Hence, numerical ratings were used as a means of precise definition only. The computed flood peaks are approximations of the peaks of floods having the definite recurrence intervals. The writers hope that the flood peaks will be understood according to these definitions.

It should be recognized that the formulas have been derived from a study of runoff from areas of less than 500 sq miles and have not been tested on larger areas. They should not be applied to drainage areas having significant physical characteristics which differ materially from those on which this study has been made, or to localities affected by storms of a different character than those in Massachusetts. Computed flood flows should be routed carefully through storage in the basin if the storage is extensive or is so located that it has an appreciable effect upon the flood peak. Before the formulas are used for areas some distance from Massachusetts, an adequate study should be made to determine their applicability. It is hoped that the method can be used in the derivation of similar formulas for other areas and, possibly, eventually for the derivation of more satisfactory formulas for general application.

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DISCUSSIONS

MILITARY AIRFIELDS
A SYMPOSIUM

Discussion

BY HIBBERT HILL, AND GAIL A. HATHAWAY

HIBBERT HILL,[34] ASSOC. M. AM. SOC. C. E.[34a]—At the request of Brigadier-General Stratton, who has been assigned to overseas duty, the writer presents the following reply to the discussers of the first paper in the Symposium.

It would appear that the design of airfields constitutes simply the synthesis of known elements into a structure of new form. The problem is a tantalizing one in that respect, for close study shows that many of the elements are also new, although seemingly familiar. It was General Stratton's purpose to reveal this point. The discussions have amply assisted him.

Loads.—Brigadier-General Newman presents a graphic illustration of the magnitude of present large airplane wheel loads, and Mr. Stanton comments in similar vein. Mr. Rudolph asks when and where the limit will be. The answer to this question is not now discernible. It does appear that, if airplane design proceeds on the basis of two main wheels carrying substantially the total load, an economic balance of some kind between cost and value of runways will soon be reached. In one instance, an estimate of cost for a single runway and taxiways, proposed for a future heavy plane, was $7,000,000. Airplane designers can extend the limit of total load, so far as it is governed by paving costs, by spreading the weight on more wheels, as Professor Casagrande suggests. In doing so, it is necessary that soils engineers and pavement engineers be consulted so that there can be a thorough understanding of the effects of tire pressure, footprint area, and wheel spacing. The airplane designer's problem, however, is to minimize the dead weight of landing gear in the interest of pay load. Landing gear for the present heavy planes represents from 10% to 15%

NOTE.—This Symposium was published in January, 1944, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1944, by James B. Newman, Jr.; March, 1944, by Thomas E. Stanton; April, 1944, by W. E. Howland, and David S. Jenkins; May, 1944, by William E. Rudolph, Raymond L. Irwin, G. G. Greulich, Hibbert Hill, Jacob Feld, and Robert Horonjeff; June, 1944, by Clarence S. Jarvis, D. D. McGuire, E. D. Parmer, A. Casagrande, and Harry E. Cotton; September, 1944, by D. O. Murray, Thomas B. Pringle, John R. Haswell, W. J. Turnbull and William H. Jervis, Ralph R. Proctor, D. M. Burmister, Ralph Hansen, H. V. Pittman, G. R. Schneider, and H. M. Williams; November, 1944, by Ira B. Mullis, and C. F. Izzard; and December, 1944, by L. J. Mensch.

[34] Lt. Col., Corps of Engrs., U. S. Army, Office, Chf. of Engrs., Washington, D.C.

[34a] Received by the Secretary February 20, 1945.

of the gross weight which, in terms of tonnage, is a very real consideration—increasingly so as gross weight increases. The solution of this problem is concentration of loads on few wheels, with resultant high pavement costs. It has been suggested by Col. Bartley M. Harloe that the great planes of the future might be compared to ocean liners, not expected to be landed under their own power, but rather to be warped carefully into position by tugs or by aerial equivalents. Under this conception, paved tracks would be substituted for wide runways. Some new thought on the subject is certainly in order. Mr. Rudolph has suggested the possibilities of catapults, and rockets may be used for the same purpose. Both catapults and rockets would project the planes into the air. Similar means for landing them may be found in reversible propellers.

In the same vein, the runway designer should be able to take all possible advantage of the peculiarities of airplane traffic distribution in designing pavements. Mr. Pringle has defined "capacity" and "limited" operation, as used by the Corps of Engineers, to rate pavement strength and life. Mr. Stanton has referred to the reduction in pavement strength permitted by the limited number of repetitions of load on runways.

In his general discussion of the Symposium,[34b] the writer also points to possible variations in pavement thickness to meet the peculiar distribution of traffic on runways. There appear to be economies in this direction which have not been fully realized.

The reality of high impact loads, as discussed by Mr. Cotton, is not in doubt. The effect of these loads on the pavement, however, is another question. Numerous measurements of pavement deflections, and a number of base and subgrade pressure determinations, have been made under static and moving loads. These measurements, made in the course of accelerated traffic tests, all show a marked reduction in deflection under the moving load, and a marked reduction of subgrade pressure—that is, an apparent spreading of the load—under the moving wheel as compared with the static wheel. These results, coupled with the fact that no airfield pavement failures attributable to impact have been observed, have led the Corps of Engineers to conclude that allowance for landing impact stresses in the pavement design is not warranted.

Site and Layout.—It is unfortunately true, as Professor Haswell has noted, that soil and drainage seem often to be the last considerations in site selection. The swamp and power line criterion mentioned by General Newman may not actually be contained in site selection instructions, but the combination has occurred too often to be comfortable. Exigencies such as those discussed by Lieutenant-Colonel Schneider, or the need for a location within given boundaries, often force selections which are not the best. However, the economies to be realized, in initial construction as well as in maintenance, from good site selections are startling. The cost of the aforementioned runway—an estimated $7,000,000—became less than $1,000,000 when the location was changed to one having a suitable soil. In the case of military airfields, and particularly

[34b] Colonel Hill's discussion of the Symposium was published in May, 1944, *Proceedings*, page 753, before it was realized that he would also be called upon to prepare the closing comment in General Stratton's absence.—Ed.

in the case of airfields for very heavy planes, sites can be sought over a wide area. An additional 100-mile flight is a bagatelle for such planes, whereas the savings of time in construction and in construction and maintenance costs, resulting from a choice location, are decisive. There is also the possibility of improved atmospheric conditions and, hence, of improved flying conditions, which may be gained from the selection of sites within a wide area.

General Newman has given a number of layout criteria for military airfields. The criteria he gives are for training fields. Under operational conditions, with trained pilots, these standards may be reduced. These criteria are not fixed. They are being studied in the light of past experience and in the light of the requirements of the newest planes. They will no doubt be continually revised as airplanes change and experience accumulates. The entire question of requirements for airfield sites and layouts appears to offer a fruitful field for continuous study.

Drainage.—The numerous comments on the critical importance of adequate drainage serve to emphasize a subject which General Stratton in no way meant to minimize. Captain Parmer cites a number of considerations of perhaps controlling interest, which indicate some of the difficulties of the subject. Mr. Feld states that pavements are not impervious at joints and cracks and that, as a consequence, positive subsurface drainage is necessary. Such drainage is automatic if base and subgrade are permeable. If base and subgrade are not permeable (the usual case), the damage is done before any feasible method of subsurface drainage becomes effective. Either the pavement must be designed to carry the load under the worst conditions, or it must be kept properly sealed. The latter is a feasible course. Mr. Greulich fears that moisture accumulation makes guesswork of plate bearing data and California bearing ratio (CBR) test information. The designs of the Corps of Engineers are based on the important assumption that base and subgrade will become saturated by capillary action and condensation which cannot be controlled by drainage. The validity of this assumption is proved in general by available data, although a serious doubt remains as to whether saturation occurs in arid and semiarid regions. In this respect, Mr. Murray makes a very interesting comment on the entire absence of any moisture accumulation under conditions in Australia.

In general terms, the function of drainage is to prevent the entrance of moisture into load bearing areas of soil. The removal of rainfall by proper surface drainage thus would be better practice, than, as suggested by Mr. McGuire, the removal of rainfall, as filtered water, by subsurface drains.

Mr. Cotton advocates a combined surface and subsurface drain. Experience with such drains has been unfortunate. If the drains are properly constructed to avoid infiltration of the surrounding soil, they quickly clog at the surface and thus become ineffective for removal of surface runoff. When drains are constructed of coarse material, their use has, in numerous cases, resulted in failure of the pavement edge because of infiltration of soil from the subgrade or base course. The drains, under these conditions, also are very likely to clog, with consequent saturation of the subgrade by backwater.

Mr. Cotton's discussion of backfill filters for subdrains is valuable. It should be emphasized that the material in Table 10 represents a maximum size.

Finer gradations, containing materials passing the 150-mesh sieve, will be required for silt and clay soils. Filters should be designed to fit the foundation soils at the site. The design of satisfactory filters is discussed in detail in the *Engineering Manual* (23),[34c] and Karl Terzaghi, M. Am. Soc. C. E., has also presented a method (63) of design.

Messrs. Cotton, Greulich, Haswell, and Feld have criticized the design of drain shown in Fig. 8(*b*). Professor Haswell's question as to the need for the natural earth backfill beneath the drain is very pertinent. It is no doubt best practice to trench exactly to grade and to lay the pipe on undisturbed soil, thus avoiding tamping beneath the pipe and the excavation required for the fill. The placing of the perforations upward, criticized by Mr. Cotton, is the result of mature consideration. With upward perforations, the hydraulic characteristics of the pipe are believed to be improved; there is not the same opportunity for outward flow in areas where ground water is below the pipe that there is when the holes are downward; and placement of the backfill and filter is facilitated. Specifications require that the filter be designed to fit the local soil conditions, so that the filter material cannot drop through the perforations. If the filter material could drop through the perforations, it could also be pushed through the perforations by hydraulic pressure in Mr. Cotton's design. Mr. Feld raises a very practical point of construction. He is entirely correct for the drain design that he discusses, but he does not consider filled shoulders, which would introduce a somewhat different problem. His remarks also recall the problem of drainage during construction. This problem is intimately related to construction procedures and is of such critical importance that, in the writer's opinion, the necessary procedures should be incorporated in the plans and specifications by the designer, and should not be left to decision in the field under the stress of construction exigencies.

Mr. McGuire expresses a hope that a clear and usable analysis of subsurface drainage requirements, comparable to Mr. Hathaway's analysis of surface drainage, will be prepared, and Mr. Greulich comments in similar vein. The writer, unfortunately, can only echo the hope. Such an analysis is greatly needed. A beginning has been made in the *Engineering Manual* (23), on the basis of available information, and revisions of the *Engineering Manual* will be made as knowledge accumulates.

Base and Subgrade Requirements.—Specifications for base and subgrade materials are intimately concerned with moisture and drainage, as Mr. Cotton notes. Mr. Cotton's remarks on the value of pervious materials from this standpoint are reinforced by the information given by Messrs. Casagrande and Parmer on the strength and compaction of such materials. Field experience is rapidly accumulating to show that a wide variety of granular materials can be compacted satisfactorily and that when so compacted these materials form very strong bases, relatively or entirely free from moisture troubles. These properties are especially important in the foundations of runways to carry the very heavy planes. Such loads create important soil stresses to depths as great as 5 ft below the surface of flexible pavements and will cause consolidation

[34c] Numerals in parentheses, thus: (23), refer to corresponding items in the Bibliography at the end of the Symposium, and at the end of discussion in this issue.

to such depths, as well as high shearing stresses at somewhat lesser depths. Recent field experience indicates that sands not fully compacted will compact under traffic without shear failure and that moisture content is not critical. On the other hand, tests (unpublished) by staff members of the Flexible Pavement Laboratory, U. S. Waterways Experiment Station, Vicksburg, Miss., show that the shearing strengths of plastic soils, including those of soils of medium and low plasticity, are very likely to decrease severely upon further compaction under traffic and that, in such a case, water content is extremely critical. Ideally, of course, any base and subgrade should be initially compacted to at least the ultimate density it will acquire under traffic. Allowing for a margin of error, however, and in view of the great wheel loads involved, the use of granular base and subgrade materials provides a comforting factor of safety.

Mr. Mullis' remarks on the inadequacy of the standard AASHO test are certainly correct, for these conditions. The modified test used by the Corps of Engineers results in a much greater density than does the standard test. It was found in tests at Barksdale Field, La., with a 50,000-lb wheel load, that compaction of from 103% to 105% of modified AASHO density was required in the 17 in. of base material (to 20 in. below the pavement surface); the compaction requirements then decreased with depth to 90% of modified AASHO density in the subgrade at a depth of 53 in. below pavement surface.

It appears beyond question that the high densities and deep compaction required by airports will necessitate the development of new equipment for this purpose, including the vibratory equipment foreseen by Professor Casagrande.

The subject of frost effects, briefly discussed by Messrs. Casagrande, Feld, and Parmer, is most important. The reasonably satisfactory treatments that have been developed in highway practice require reexamination and refinement in the light of the deep and expensive base construction required for airfield loads. In this respect, again, the value of clean granular base courses, not subject to frost effects, is reiterated. The possibilities of such insulating materials as Professor Casagrande mentions, and the possible advantage that can be taken of relative heat transmission coefficients, as noted by Mr. Feld, are appreciated. In the past, reliance has been placed on natural granular materials—in large part because these materials have been available and furthermore because of a lack of information on the behavior of other materials. Curiously enough, in areas of permanently frozen ground mentioned by Professor Casagrande, the principal problem is most often one of avoiding interference with the natural ground-water regimen rather than one of maintaining a permanently frozen subsoil. For this purpose very coarse granular materials are best.

Construction Practices.—The discussion by Mr. Pittman emphasizes the fact that design and construction practices must be coordinated to obtain a satisfactory job. He and Mr. Feld agree that successive lane construction of concrete pavements is the best practice. In the writer's opinion, this method of construction is essential for good results, unless someone could guarantee that there would be no rain! The writer also agrees that the usual loss of bearing power under thickened edges, resulting from excavation of the compacted base

material, is likely to offset the value of the thickened edge, as noted by Messrs. Feld and Parmer. The same excavation traps rain water during construction, thus also contributing to the loss of bearing. For these reasons, the Corps of Engineers has eliminated the thickened edge as standard practice, except at exterior edges and at the longitudinal expansion joint at the center of the runway. Emphasis has been placed on obtaining compaction and base support under these edges equal to the design assumptions.

Mr. Pittman calls attention to a very troublesome design feature found in those cases where the pavement is below ground level at the shoulders. Thus, subgrade and pavement have to be constructed in a broad shallow trench. If the soil is not pervious, this situation results in drainage difficulties during construction and quite possibly after construction. In this case, design and construction sequences must each be directed toward providing means of drainage during and after construction. Mr. Pittman gives one solution.

Mr. Pittman's remarks on winter construction arise from extensive experience. It is no doubt correct that concrete pavements can be constructed under conditions that would make construction of flexible pavements impossible. The writer believes, however, that, unless the need is pressing, as it has been during World War II, airfield construction should not be undertaken in seasons of unfavorable weather. In the case of such construction, extremely conscientious attention must be given to the quality of work, and delays must be accepted, to obtain desired results. Thus, an almost impossible burden is placed on contractors and inspectors. It may be remarked, also, that one disadvantage of concrete is that it can be placed on an unsatisfactory base, to look well on the surface but in fact to have nowhere near the load supporting value contemplated in the design. The base for flexible pavement must have strength enough, at least, to permit rolling of the pavement surface!

Mr. Proctor advocates compacting the soil to about a foot above grade, then trimming this foot off to reveal a fully compacted surface. In the last analysis, the question of how to obtain the desired compaction at the surface is an economic one. Subgrade planers are used to advantage to remove the top inch or so, in both highway and runway construction. However, it is the writer's opinion that the removal of 12 in., as proposed by Mr. Proctor, is not essential and is not economical. Mr. Proctor's remarks on the use of the sheepsfoot roller, and on emergency construction procedure, are very interesting, and grow out of long experience. These remarks center around the use of an "available" roller. To obtain the high compaction necessary in airfields for heavy planes, it will be necessary to specify the use of particular rollers, although admittedly the characteristics of those rollers are not yet fully determined.

The use of membrane curing for concrete pavements is permitted by specifications of the Corps of Engineers. These specifications require moisture retaining ability as determined by an established test procedure. This method of curing has great advantages in ease and speed of application, economy, and negligible maintenance cost. The material has no insulating or cooling properties, and it may be, as Mr. Pittman states, that its use in hot weather will permit dangerous thermal stresses in the concrete to occur, especially if the material is of dark color. Mr. Pittman's statement, however, is the first to that

effect which has come to the writer's attention, and the writer has been unable to find any case in which damage to the concrete can be ascribed to such thermal stresses.

Design.—Messrs. Turnbull and Jervis, and Hansen have presented interesting background material on the development of the CBR method and on the method of accelerated traffic testing. Their statements with respect to the findings of these investigations are just sufficient to whet the reader's interest, but by no means to satisfy it. In continuation of their remarks, it may be stated that, since Mr. Hansen's discussion was written (September, 1944), the Barksdale tests have been made available to a limited distribution (64).

The application of the formulas introduced by H. M. Westergaard, M. Am. Soc. C. E., to design of pavements on marshy ground is questioned by Mr. Horonjeff. His doubt, it appears to the writer, is not in fact directed at the formulas but rather at the method of determining K which is used by the Corps of Engineers. In the writer's opinion, Mr. Horonjeff's question is justified. The plate bearing test for the determination of K has little to recommend it except that no other method is available and except that, within the usual range of subgrade soils, it gives results which are reasonably consistent with experience. There is no present method for extending this experience to soils such as Mr. Horonjeff describes. On such soils, the plate bearing test will give indeterminate results; yet such soils do have supporting values under rigid slabs.

Professor Casagrande's remarks on the difficulties of a theoretical approach to the design of flexible pavements are illuminating. In spite of these difficulties, engineers will not rest until such an approach has been accomplished. Present studies by the Corps of Engineers are aimed at computing the soil stresses, and specifying the necessary treatment of soils and bases to resist these stresses. The accomplishment of this goal is an indefinite distance in the future. Until that time, refinement of the CBR and similar empirical methods will not be neglected.

Messrs. Casagrande and Horonjeff both discuss the problem of load transfer between rigid slabs under heavy wheel loads. This problem becomes more pressing as loads and slab thicknesses increase. One absolute requirement for a successful joint is that the joint surely function. In this respect, the writer fears the doweled joints, suggested by Professor Casagrande, in which the dangers of improper installation would be increased by the large number of joints.

In the writer's opinion, there is no such value as an "allowable pavement deflection," on which Professor Burmister hopes to obtain factual information. Considering the pavement alone, the allowable deflection is a function of the curvature, and will be greater the longer the radius of curvature; hence, the allowable deflection will be greater the larger the tire imprint—other factors being equal. Allowable deflection will also vary with the soil and base constants. In the Barksdale tests, initial subgrade deflections, exceeding about 0.1 in. under moving wheel loads of 50,000 lb, increased under succeeding wheel passages to more than 0.2 in., and shear failure occurred. In similar tests with 40,000-lb wheel loads, but on a pavement laid over a fine sand subsoil, no failure

occurred although the initial deflections were somewhat greater than at Barksdale. Considerable settlement did occur, however, as the subsoil compacted. In such cases, the differential, rather than the maximum, settlement is important. To the writer's knowledge, no flexible airfield pavement has been overstressed as the result of compaction, unaccompanied by shear failure, under traffic. T. A. Middlebrooks and R. M. Haines, Assoc. Members, Am. Soc. C. E., have presented the results (65) of numerous deflection measurements, and Mr. Stanton also contributes some information.

In Table 13, Professor Burmister introduces a thickness factor which, when applied, permits a good base material to be less thick than a poor one. The CBR method of design is predicated upon the hypothesis that a given total thickness of base and pavement is required over a given subsoil or base and that the total thickness is independent of the quality of the material as long as the material has sufficient strength. This hypothesis is based upon experience, and is definitely proved by the Barksdale and other similar tests. Professor Burmister thus argues erroneously that, because material with a CBR value of 50 is allowed immediately under pavements for light wheel loads, a lesser thickness of material with a CBR value of 80 would be suitable. These minimum CBR values are needed to resist stresses (shear principally) in the base material itself, and do not influence materially the stresses in the underlying soil, so far as available evidence shows. This statement must be limited to the usual base materials. Evidently, in the extreme, it would not apply to concrete.

The general adoption of relative density in place of percentage compaction, as suggested by Professor Burmister, is an end to be desired. It has special virtue in the case of sandy soils, since these soils undergo marked changes of strength in passing from loose to dense states. However, in order to use relative density as a measure, it is necessary to have some standard definition of the end points—that is, of the loose and dense states. Such a definition or test is not available.

Mr. Proctor, raises a number of interesting questions concerning soil compaction in earth dam construction. In reply to one of his first statements, it should be mentioned that methods of compaction and compaction control developed in earth dam practice are the bases for similar operations in pavement construction. It does not follow, however, that criteria which have been successfully applied in earth dam construction will suffice for successful pavement construction. The maximum unit loads found in earth dam construction greatly exceed the unit loads encountered in pavement construction, but this comparison does not consider the size of the loaded areas, the load distribution, or the repeated stresses found in pavement practice. Because of these factors, all present information leads to the conclusion that the permissible soil strain in pavement practice is only a fraction of that required to develop the ultimate strength of the soil under a single loading, and permitted in earth dam practice. It appears, therefore, that, for airfield pavements, relatively higher densities must be obtained than are commonly accepted in earth dam practice. It appears also that, possibly excepting sands, the bases and subgrade must be initially compacted to substantially such a density that no further compaction

will occur under traffic. On the basis of the same information, the writer very much doubts Mr. Proctor's interpretation of Fig. 41, although all the information necessary to resolve this doubt is not given in Fig. 41.

To clarify several apparent misapprehensions which appear in Mr. Proctor's discussion, it may be stated, first, that the CBR method is not dependent upon an optimum density. This test evaluates the soil at any density and moisture content. The most desirable values of density have been discussed in previous paragraphs. On the basis of available tests, it appears that the most desirable water content is somewhat on the dry side of optimum for the compactive effort applied.

Secondly, Mr. Proctor very correctly points out the great reduction in strength of soil at 90% compaction, as compared to its strength at 100% compaction. It should be noted that 90% of modified AASHO density is still a rather high density by usual standards, but, more especially, that 90% compaction is allowed by the specifications of the Corps of Engineers only in fill, the top 6 in. of which are required to be compacted to 95% density. The 90% criterion is not applied to base courses. The present hope is to determine the actual density and compacting moisture content necessary at a given depth, and to specify the values. In the aforementioned Barksdale tests, it was determined that 90% compaction was sufficient at certain depths, 95% was required at an intermediate depth, and from 103% to 105% was needed in the upper base materials.

In the same connection, Mr. Proctor stresses the need for control of moisture content at such a value that subsequent consolidation, from whatever cause, will not build up excess hydrostatic pressures in the soil. This is a very grave possibility in airport practice where the heavy wheel loads may cause consolidation under traffic, and, if such pressures occur, they are fatal to the pavement. This phenomenon was observed in one failure of a test pavement on a silty clay soil.

Messrs. Rudolph and Horonjeff each briefly raise the problem of reinforcing existing pavements to withstand heavier loads than those for which they were originally designed. The problem has already arisen at a considerable number of Army fields. The procedure has been first to evaluate subgrades, bases, and pavements as they actually exist, by field and laboratory tests—and, where applicable, by accelerated traffic tests—then to design the reinforcement. The thickness of flexible pavements is increased in accordance with the CBR requirements. This increase may involve base courses on top of the existing pavement. In such a design, careful consideration must be given to the effects of moisture in the new base; hence, closed crushed stone is preferred for this base. In the case of concrete pavements, the thickness of a slab required for the new and greater load is calculated, and the equivalent two-beam (one beam being the existing slab) thickness is determined. The general practice thus far has been to place a leveling course of sand asphalt, where necessary, or a thin bituminous bond breaking course, between the old and new slabs. This is done to relieve temperature stresses in the upper slab, but principally to avoid irregular bearing of the upper slab following movements resulting from temperature effects.

Present limited experience with such designs is satisfactory. When pavements are thus thickened, original grades and drainage may be so disturbed that the required changes in these elements result in an uneconomical over-all design. It may then be necessary to remove and reconstruct a part of the existing pavements to meet grades at buildings and drainage structures.

In conclusion, it is evident from the discussions, and from General Stratton's paper, that, although much progress has been made toward the rational and exact design of airfield pavements, many questions are at present unanswered, and much remains to be done. Mr. Pringle has outlined a program of periodic condition surveys, and other discussers have mentioned the need for correlation of test and laboratory results with field experience. A number of organizations are working on these problems. The Corps of Engineers has established three laboratories to work continuously on the correlation of field and laboratory experience and on the extension of knowledge through field testing. These laboratories are the Flexible Pavement Laboratory at Vicksburg, Miss., the Rigid Pavement Laboratory at Cincinnati, Ohio, and the Frost Effects Laboratory at Boston, Mass.

GAIL A. HATHAWAY,[35] M. AM. SOC. C. E.[35a]—The comments and suggestions on drainage facilities contained in the discussions of the Symposium are appreciated. Several valuable modifications in procedure have been suggested, and recognized deficiencies in the method have been indicated.

Professor Howland presents a mathematical method for the determination of a runoff hydrograph for a rain of limited duration and compares the hydrograph so obtained with hydrograph No. 2, Fig. 18. The rising limb of hydrograph No. 2, Fig. 18, was computed by the general equation (Eq. 2) developed by Mr. Horton and utilized by Professor Howland, whereas the crest shape and recession limb of the hydrograph were derived from the curve of overland flow rates by utilizing the unit hydrograph procedure. The unit hydrograph method is recognized as an empirical procedure based on the analysis of actual runoff data. However, the writer has found that the method gives excellent results for small areas when reliable and accurate basic information is available. The limitations of the equation developed by Mr. Horton for lengths of flow greater than 600 ft are recognized, and alternate procedures are contained in Mr. Williams' discussion. In that connection, a few test runs by Mr. Izzard on the bluegrass turf strip, in which instance rainfall was stopped before equilibrium was reached, would have furnished valuable information on the shape of the crest and recession limb of the hydrograph. Professor Howland's contribution outlines problems as yet unsolved and emphasizes the need for actual runoff data from airfields.

The writer agrees with Mr. Jenkins' statement stressing the need for, and the practical value of, a table showing the range of infiltration capacities for different soils and soil covers. The data on infiltration capacities as shown in Tables 6 and 7, and in Mr. Jenkins' accompanying discussion, are very informative and valuable for design purposes. More information of a similar nature

[35] Head Engr., Corps of Engrs., Office, Chf. of Engrs., Washington, D. C.

[35a] Received by the Secretary March 26, 1945.

in the files of the Department of Agriculture should be made available to the engineering profession. Mr. Jenkins states that airfield turf is generally not well established until about the end of the first year, and in many cases not for two years, following completion of grading. It has been the writer's observation that many fields do not have a grass cover at the end of the third growing season although every effort has been made to secure a good turf. The points raised by Mr. Jenkins on the applicability of the unit hydrograph theory to the determination of runoff from graded airfield slopes 500 ft or 600 ft long have been covered to some extent by Mr. Williams.

With reference to Mr. Jenkins' comments on the relation of airfield grading requirements to drainage, it is gratifying to note that the importance and necessity of good drainage have resulted in modifications and improvements of grading criteria for Army airfields. The transverse slopes of runway shoulders and graded areas have been increased to 2%. Along the drain line, normally 250 ft from the runway center line, a 2% longitudinal grade is permitted with a maximum change in grade of 3%. In addition, where the foregoing criteria will not permit development of ponding areas, it is permissible to provide a small transverse ridge at least 100 ft downstream from the inlet, with the crest not more than 0.5 ft above the inlet.

Mr. Jenkins suggests that the method of maintaining the hydraulic grade line near the top of the drain pipe be clarified. The design assumptions are based on a controlled outflow from each ponding basin. He also discusses the formula used by the Civil Aeronautics Administration (see Eq. 9). The utilization of this formula in the determination of runoff from airfield areas is questionable. For example, T (the duration of rainfall) is assumed to be 1 hour. The use of a value of 1 hour for T with no consideration for varying times of concentration, particularly for apron and runway design, is obviously incorrect as the computed runoff will be inadequate for numerous high intensity storms of short duration but with a frequency less than that of the design storm. The quantity t in Eq. 9 (time allowed for removal of rainfall after the end of the storm) assumed to be 2 hours is actually a storage correction; and, in many cases, the storage is not available. It has been observed that the apparent simplicity and ease of using Eq. 9, with T equal to 1 hour and t equal to 2 hours, have led to numerous cases in which storage reductions are assumed without the required storage being available or even being considered. Mr. Jenkins' thorough discussion covers several points not touched by the writer and clarifies and expands others in the paper.

The comments of Mr. Irwin on the need for infiltration data and the results of his experience with clogging are timely and informative. Much work remains to be done in obtaining accurate field data on roughness coefficients for varying types and degrees of grass cover. In that connection, the tests conducted by Mr. Izzard are of inestimable value.

It is of interest to note that Mr. Jenkins' computations for a typical layout using the modified rational method (Eq. 9) give runoff values considerably lower than those obtained by the procedures described by the writer, whereas the example presented by Mr. Irwin gives somewhat higher runoff values. The writer concurs in Mr. Irwin's statement that a careful study of critical

rainfall distribution deserves investigation to determine critical conditions for the area under consideration.

Mr. Greulich and Mr. McGuire stress the great need for a type of analysis that can be utilized in the design and efficient adaptation of subdrainage pipe. More data are needed on this phase of airfield drainage, and every effort should be made to conduct field tests supplemented by the necessary laboratory research to provide a satisfactory method for rational design of subsurface

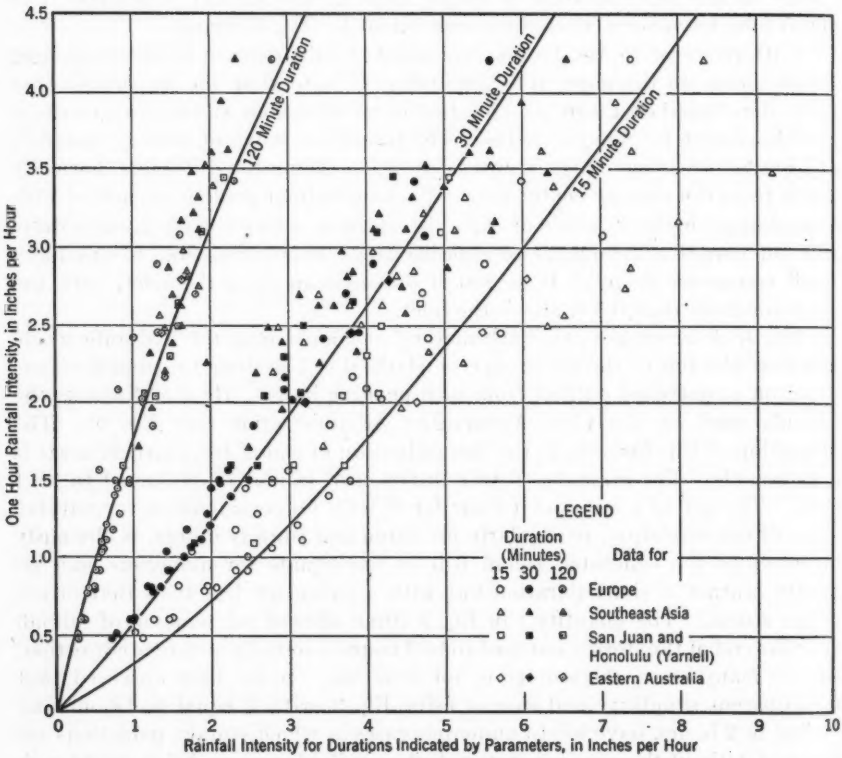

FIG. 54.—RELATION OF 1-HR RAINFALL INTENSITIES TO INTENSITIES FOR DURATIONS OF 15, 30, AND 120 MIN IN CERTAIN FOREIGN COUNTRIES

drainage systems. It is desirable to maintain a low ground-water table in the airfield area, and lowering the ground-water table can be facilitated by providing proper surface drainage.

The theoretical purpose of the "surface intercepting drain" or modified "French drain" (namely, to intercept the surface water without changing the uniform sheet flow to a piled-up flow) is a good one; but unfortunately the intercepting drains fail to function, in many instances, under practical operating conditions. Not only do the surface intercepting drains become silted by fines but, in most instances, the assumed collecting capacity is in excess of the quantity of water that is intercepted by the drain. Instances have been

recorded in which the pavement base-course material has moved into this type of drain, resulting in the subsidence and failure of the outer lanes of the runway. The suggested combined catch basin and intercepting drain for turfed valleys in Fig. 39 may have considerable merit for ponding areas, provided the intercepting drain does not permit water to have access to the foundation of the runway or apron pavement.

The statement by Mr. Murray that intensity curves for rainfall on the eastern coast of Australia conform closely to those in Fig. 15 is especially significant. Subsequent to the preparation of the curves in Fig. 15, rainfall intensity data were obtained for Europe and southeast Asia which conform closely to those for the United States. (Data for eastern Australia were taken from Table 12. In foreign countries, data for durations of 5 min and 240 min are not available.) These data have been plotted in Fig. 54 for comparison with the curves previously developed in Fig. 14 for the United States. In general, the foreign data for short-duration rainfall indicate a lower intensity—a condition which may possibly be attributed to a lack of recording gage records. However, the general agreement of the foregoing data does suggest a possible universal application of the design methods discussed herein.

The discussion by Mr. Williams is a valuable contribution to the paper, and the procedure he suggests will be found useful in extending the design criteria to the determination of runoff from larger airfield areas.

Mr. Izzard's comparison of actual hydrographs resulting from simulated rainfall with the theoretical hydrographs included in the paper provides a valuable check on the accuracy of the runoff computations. Although a shift in the time axis is necessary to bring the adjusted hydrograph into position with the actual hydrograph, the change would not seriously affect the ultimate drain-inlet capacity. Laboratory and field checks along the lines pursued by Mr. Izzard are of inestimable value in arriving at correct design procedures and in establishing confidence in new and better approaches to the solution of rainfall-runoff problems.

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(23) *Engineering Manual*, Constr. Div. Office of Chf. of Engrs., U. S. Army, 1943. (*a*) Chapter 21, Design of Drainage Facilities.

(63) **"Stability and Stiffness of Cellular Cofferdams,"** by Karl Terzaghi, *Proceedings*, Am. Soc. C. E., September, 1944, p. 1037.

(64) **"Service Behavior Test Section, Barksdale Field, La.,"** Corps of Engineers, U. S. Army, U. S. Engr. Office, Little Rock, Ark.

(65) **"Results of Accelerated Traffic Tests of Runway Pavements,"** by T. A. Middlebrooks and R. M. Haines, *Proceedings*, Highway Research Board, National Research Council, 1943, p. 101.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

LOCK MANIFOLD EXPERIMENTS

Discussion

BY ROBERT F. KREISS, AND RALPH G. BERK

ROBERT F. KREISS,[18] JUN. AM. SOC. C. E.[18a]—Manifold tests whose results are in good agreement with the discharge distribution and with the over-all coefficients computed from calibrations of a single port confirm the usefulness of port calibrations. Inspection of the coefficient curves of Fig. 6 suggests that a plot of port coefficients against the ratio of average port velocity to conduit velocity would be significant in interpolating test results to ports of other sizes. Rather than present graphs of such relations, the writer has computed the characteristics of discharging manifolds from the calibrations on Ports I to VII. Two characteristics are shown as functions of the ratio of cumulative port area to conduit area in the semilogarithmic plots of Fig. 19. The results, which are the practical objectives of the single port calibrations, indicate that for ports of similar design the relative distribution of discharge and the over-all coefficients for manifolds are nearly single valued functions of the area ratio (compare results for Ports I to III and for Ports IV to VI). The discontinuity in the coefficients of the single port calibrations is apparent in the coefficients for the manifold, the discontinuity for Ports IV to VI occurring at an area ratio of about 0.4.

The computations for the manifold have been made essentially as outlined by the authors. The over-all coefficients assume a manifold acting as a unit such that the head producing discharge Q_1 through N ports of area A_3 is $H_d + \frac{(V_1)^2}{2g}$, or

$$C = \frac{Q_1}{N A_3 \sqrt{2g} \sqrt{H_d + \frac{(V_1)^2}{2g}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

the discharge, velocity, and head being those immediately upstream from the most upstream port. The results in Fig. 19 represent characteristics for a frictionless conduit. The discharge distribution in Fig. 19(*b*) refers to a unit

NOTE.—This paper by Edward Soucek and E. W. Zelnick was published in October, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1945, by J. Pietrkowski; and April, 1945, by John S. McNown, F. W. Edwards, and W. M. Lansford.

[18] Engr. (Hydr.), U. S. Engr. Sub-Office, Hydraulic Laboratory, Iowa City, Iowa.

[18a] Received by the Secretary February 28, 1945.

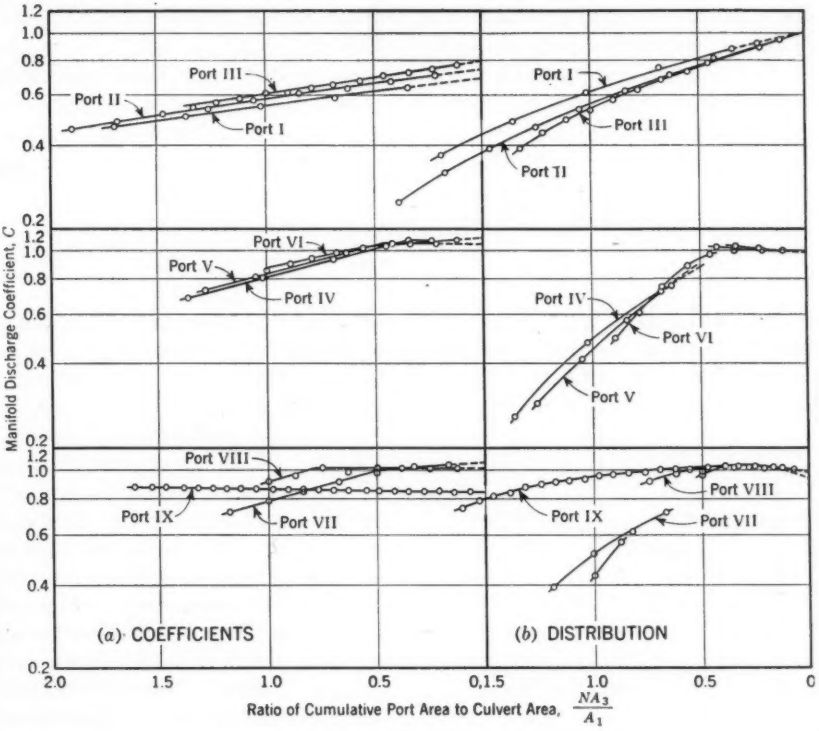

FIG. 19.—MANIFOLD DISCHARGE CHARACTERISTICS

TABLE 4.—VALUES OF C' AND C'' FOR FRICTIONLESS CONDUITS

Port	Type of Port Discharge							
	Port Flowing Full				Port Flowing Partly Full			
	C'	C''	Range of Area Ratio		C'	C''	Range of Area Ratio	
			Minimum	Maximum			Minimum	Maximum
I	....	....	....	....	0.70	0.10	0	1.7
II	....	....	....	....	0.75	0.11	0	1.9
III	....	....	....	....	0.80	0.12	0	1.3
IV	1.09 +	0 +	0	0.34	1.27	0.19	0.34	1.4
V	1.07 ±	0 ±	0	0.47	1.33	0.20	0.47	1.3
VI	1.12	0.07	0	0.55	1.26	0.16	0.55	1.0
VII	1.06	0.05	0	0.5	1.28	0.21	0.5	1.2
VIII	1.00	0.00	0	0.8	1.34	0.16	0.8	1.0
IX	0.86	− 0.06	0	1.6	....	....	....	....
X	0.96	0.06	0	1.3	....	....	....	....
XI	0.77 ±	0 ±	0	1.3	....	....	....	....

discharge either at an area ratio approaching zero as a limit (Ports I to III) or at the downstream port (Ports IV to IX).

An additional observation, worthy of note, is that the over-all coefficients, shown in Fig. 19(*a*), bear a relation to the area ratio as follows:

$$C = \frac{C'}{10^a} \quad \ldots\ldots (19)$$

in which, to simplify typography:

$$a = C'' \frac{N A_2}{A_1} \quad \ldots\ldots (20)$$

In Eqs. 19 and 20, C' and C'' are constants dependent on the port design and on the two types of discharge mentioned by the authors. Computed manifold characteristics for ports of four other designs further check the validity of Eq. 19. Results for Ports VIII and IX are shown in Fig. 19. The values of the coefficients C' and C'', the range of area ratios to which the coefficients apply, and the type of discharge from the ports are given in Table 4 for Ports I to XI. The tabulated values are subject to the limitations given by the authors (see heading, "Single Port Tests: Limitations"). It is evident that the discharge distribution and the efficiency of the manifold are dependent on the type of ports in the manifold.

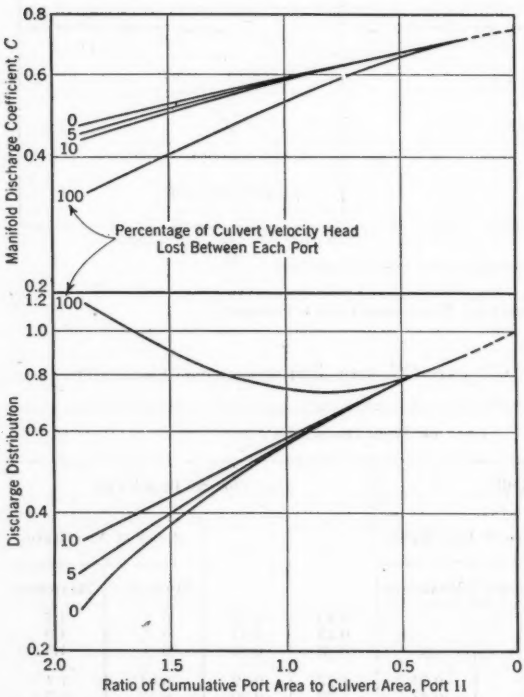

FIG. 20.—EFFECT OF FRICTIONAL LOSSES IN A CULVERT ON THE HYDRAULIC CHARACTERISTICS IN A MANIFOLD

Port VIII is similar to Port VII (see Fig. 4) except that the dimensions of the opening are 1.52 in. (parallel to the conduit center line) by 2.97 in. Port IX is a tapered flow-line port with one face extending from the floor of the conduit. Ports X and XI extend downward from the conduit floor and then horizontally, the horizontal leg being of constant shape in Port X and divergent in Port XI. (Details of design and port calibrations have been presented by Sydney O. Steinborn,[19] Jun. Am. Soc. C. E., and are on file in the Engineering Societies Library.[20])

[19] "Manifold Studies, Part III," by Sydney Steinborn, The Panama Canal, Special Eng. Div., Diablo Heights, Canal Zone, December, 1941.

[20] 29 West 39th St., New York, N. Y.

An exponential equation was reported in 1937[20] for filling and emptying operations in lock models having port designs similar to Port IX. In more recent (1944) tests on another lock model, the exponential equation was found applicable to steady flow calibrations and to varied flow operations as the number of lateral culverts (containing ports and connected to a main conduit) was varied. These results provide additional evidence of the relation that exists between single port calibrations and discharging manifolds.

The conduit friction has an appreciable effect on the characteristics of a discharge manifold when ports are widely spaced. An example for Port II is shown in Fig. 20. The conduit friction between each port is assumed as 0, 0.05, 0.10, and 1.00 of the velocity head immediately upstream of each port. (In Manning's equation these frictional values represent port spacings of roughly 0 ft, 1 ft, 2 ft, and 20 ft in a 6-in. square conduit with $n = 0.10$.)

The methods of testing and analysis presented by the authors are suitable for such designs as lock manifolds, storm sewers, sprinkler systems, and penstocks from a feeder line. With the graphs in Fig. 19 the design of manifolds, when ports are closely spaced, is further simplified and clarified.

RALPH G. BERK.[21,21a]—A valuable method of predicting the hydraulic performance of lock manifolds is presented in this paper. The method can be applied to other types of manifolds, such as filter, bottom-lateral systems, sprinkling filter underdrain systems, irrigation piping distribution systems, and orange orchard, smudge-pot fuel-supply piping.

The authors' procedure is similar to the method employed for the calculation of backwater curves in open channels. Orifice coefficients are used for calculating the flow through each port in the manifold. These orifice coefficients are derived from data obtained in testing single port models. The test results are expressed by curves showing the variation of the orifice coefficient C with changes in the value of P (ratio of flow through the port to flow in the conduit).

The orifice coefficients derived by the authors differ from the orifice coefficients ordinarily given in standard textbooks on hydraulics. The former reflect the inequality in flow through the ports of a manifold caused by abrupt changes in pressure and velocity of flow along the manifold tube, and by angularity of port discharge. None of these effects can be directly evaluated. Under the heading, "Single Port Tests: Analysis," the authors state that their derived orifice coefficients show no significant dependence upon variables other than P.

Despite the fact that the (C-P)-curves in Figs. 6 and 7 are reduced to a very small scale, it is still possible to observe that these curves assume different relative positions for varying sizes of ports. In Fig. 21 the same data are presented in a form designed for easier comparison. The (C-P)-curves for the square-cornered ports (Fig. 21(a)) indicate a consistent variation of C with the different sizes tested. This variation most likely conforms to the model laws expressed by Froude's criterion of dynamic similitude. However, the (C-P)-curves for round-cornered ports (Fig. 21(b)) show no consistent variation of C with the sizes tested. The shapes of these curves and the juxtaposition probably indicate no conformity with Froude's criterion.

[20] "Laboratory Tests on Hydraulic Model of Filling and Emptying System for Proposed Watts Bar Project Lock," U. S. Engr. Sub-Office, Iowa City, Iowa, December, 1937.

[21] Designing Engr., Water Section, with Cons. Engrs., Parsons, Brinckerhoff, Hogan & Macdonald, Caracas, Venezuela.

[21a] Received by the Secretary January 26, 1945.

In view of these differences, can one conclude that the geometrical shape of the port, together with variation in size, exercises some influence on the port orifice coefficient? Can tests with model ports of smaller scale provide enough information to permit the formulation of a model law defining the performance of a port of given shape in terms of its size? Discussing a specific example,

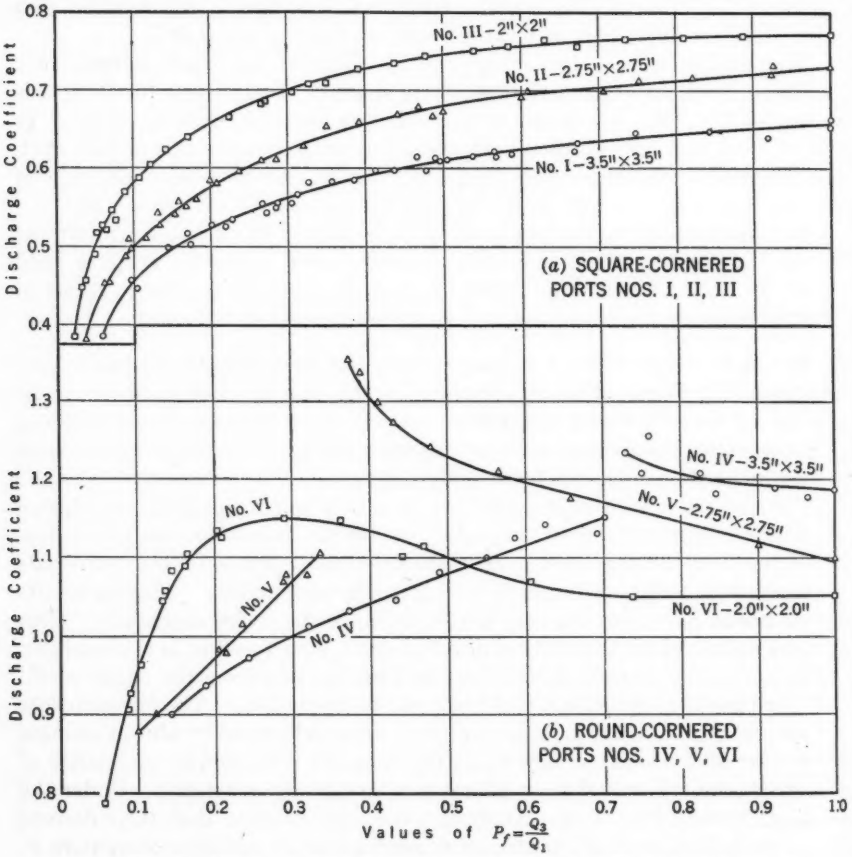

FIG. 21.—CURVES OF PORT CHARACTERISTICS, PORTS I TO VI, INCLUSIVE

what values of C for round-cornered ports (Ports IV, V, and VI) would the authors use in calculating the performance of a prototype lock manifold, with ports which should necessarily be much larger than the test ports? (The example shown in the paper is a calculation of the performance of a manifold having ports of the same size as those of the test model.)

To summarize, the writer believes that the authors do not present definite information for solving the practical problem of designing a prototype manifold larger in size than the test models and with port shapes other than square cornered. He must also conclude that, for purposes of practical design, it would be best to conduct tests on ports of prototype dimensions and shapes.

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DISCUSSIONS

SHEAR AND BOND STRESSES IN REINFORCED CONCRETE

Discussion

BY STANLEY U. BENSCOTER AND SAMUEL T. LOGAN

STANLEY U. BENSCOTER,[23] AND SAMUEL T. LOGAN,[24] JUNIORS, AM. SOC. C. E.[24a]—The discussions provide a number of interesting and useful additions to the solution of the problem presented in the paper. It has been properly noted in several discussions that the definition of $j\,d$, as given by the writers, is not strictly correct. However, as a matter of convenience, the definition is commonly used in design practice. Professor Peabody has correctly criticized the writers' misleading statement regarding the validity of the standard shearing stress formula in members without thrust having compressive steel.

Several discussers have correctly noted that the neutral axis is not parallel to the reinforcement as shown in Fig. 2. Messrs. Aleck, Ferguson, and Bergman have developed formulas based on the slope of the neutral axis with respect to the reinforcement. It should be noted carefully that the formulas proposed by the writers do not include any effect of the rate of change of flexural properties along the member. Consequently, the factors that enter into the formulas, and the development of the formulas, can be completely illustrated by a figure which neglects the slope of the neutral axis.

The application of the classical shearing stress formula to reinforced concrete, as given by Eq. 3, can be developed in undergraduate classrooms in a manner such as that used by the writers. In the graduate classroom a more comprehensive development can be given as illustrated by Eq. 19. This formula considers the rate of change of the flexural properties along the member. An additional term could be obtained by regarding the thrust P as a variable in order to obtain a formula applicable to arches.

Some discussion of Eq. 19 may be worth while. If all terms except the first are dropped, the writers' formula is obtained. It appears that the additional

NOTE.—This paper by Stanley U. Benscoter and Samuel T. Logan was published in March, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1944, by F. R. Shanley, B. J. Aleck, Dean Peabody, Jr., and William E. Wilbur; June, 1944, by L. E. Grinter, and Anders Bull; and September, 1944, by Victor R. Bergman, and Phil M. Ferguson.

[23] Associate Engr., Bureau of Aeronautics, Navy Bldg., Washington, D. C.

[24] 1st Lt., Corps of Engrs., U. S. Army, Vicksburg, Miss.

[24a] Received by the Secretary March 14, 1945.

terms beyond the first term are more or less compensating. This is suggested by the fact that calculations presented by Mr. Bergman, based on a consideration of the rate of change of k along the member, agree well with the values computed from the writers' formulas. Thus, a dilemma which the writers are unable to resolve is presented. Why does a method of analysis which considers the rate of change of flexural properties along the member give values which agree with those computed from a formula which does not consider the rate of change of flexural properties?

For further illustration of this dilemma, consider Eqs. 28 and 29. For the member without compressive steel the formula for B becomes

$$B = \frac{\frac{1}{2} - \frac{k}{3} - \frac{j}{2}}{\frac{1}{2} - \frac{k}{3} - \frac{e}{d}\left(\frac{1}{2} + \frac{n\,p}{k^2}\right)} \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

The quantity $(1 - B)$ becomes

$$1 - B = \frac{3\,dk^2 - dk^3 - 3\,e\,k^2 - 6\,e\,n\,p}{3\,dk^2 - 2\,dk^3 - 3\,e\,k^2 - 6\,e\,n\,p} \ldots\ldots\ldots\ldots (60)$$

When $P_c = 0$, Eqs. 22 become

$$P\,e = b\,d^2 f_c \frac{k}{2}\left(1 - \frac{k}{3}\right) \ldots\ldots\ldots\ldots\ldots\ldots (61a)$$

and

$$P = b\,d\,f_c\left[\frac{k}{2} - \left(\frac{1}{k} - 1\right) n\,p\right] \ldots\ldots\ldots\ldots\ldots\ldots (61b)$$

Dividing Eq. 61*a* by Eq. 61*b* gives

$$e = \frac{dk^2\,(3 - k)}{3\,k^2 - 6\,(1 - k)\,n\,p} \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Substituting Eq. 62 into Eq. 60 gives

$$1 - B = \frac{2\,(k - 2)\,(k - 3)\,n\,p}{k^3 + 4\,(k^2 - 3\,k + 3)\,n\,p} \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

If Eq. 63 is substituted into Eq. 29, the resulting formula for v will be the same as Eq. 31. Thus, Eq. 30 is in error, and the computations, based on Eq. 30, lead to incorrect conclusions. It is especially worthy of note that a formula can be derived, based on the rate of change of k and j along the member, which is exactly the same as the writers' formula which does not consider the rate of change of flexural properties along the member. This dilemma must be resolved by future research.

A few comments on formulas involving k and j rather than the section modulus may be in order. Just as in the case of tapered beams without thrust,[25]

[25] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, Am. Soc. C. E., Pt. 2, June, 1940, Eq. 5*a*, p. 56.

the shearing stress may be calculated from the formula,

$$v = \frac{V_1}{b\,j\,d} \quad \ldots\ldots (64)$$

in which V_1 is a reduced shear. The beam column of reinforced concrete is prismatic externally but nonprismatic internally. Although the surface elements are parallel, thus giving the member a prismatic shape externally, the resultant internal tensile and compressive forces are not parallel.

The reduced shear V_1 is calculated from the total shear V by deducting a reduction factor. This reduction factor is proportional to the tangent of the angle δ_1 made by the internal compressive force with a normal to the cross section. Thus, V_1 may be written as

$$V_1 = V - C \tan \delta_1 \quad \ldots\ldots (65)$$

The validity of Eqs. 64 and 65 may be demonstrated readily by using Eq. 2 and referring to Fig. 14(*a*). Eqs. 64 and 65 correspond to Eq. 21, Eq. 43, or

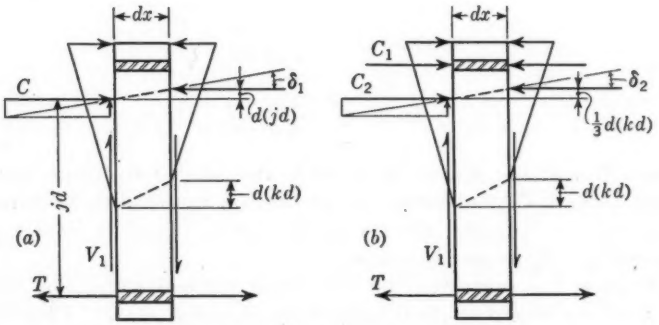

FIG. 14.—SHEAR REDUCTION FACTOR

Eq. 46. The compressive force C can be expressed in terms of k. The quantity $\tan \delta_1$ can be expressed in terms of derivatives of k and j in various ways such as,

$$\tan \delta_1 = d\,\frac{dj}{dx} \quad \ldots\ldots (66a)$$

$$\tan \delta_1 = d\,\frac{dj}{dk}\,\frac{dk}{dx} \quad \ldots\ldots (66b)$$

$$\tan \delta_1 = d\,\frac{dj}{de}\,\frac{de}{dx} \quad \ldots\ldots (66c)$$

$$\tan \delta_1 = d\,\frac{dj}{dk}\,\frac{dk}{de}\,\frac{de}{dx} \quad \ldots\ldots (66d)$$

or

$$\tan \delta_1 = \frac{V}{P}\,d\,\frac{dj}{dk}\,\frac{dk}{de} \quad \ldots\ldots (66e)$$

Eq. 66*e* has been used by Professor Ferguson. The factor $-\frac{dj}{dk}$ is given as R

in Eq. 51. The product $d\frac{dk}{de}$ is given as B in Eq. 54. The first factor is a function of k only, whereas the second factor is a function of both k and e. These derivatives are properties of the cross section and are rather complicated. By a simple expedient the first factor may be eliminated.

Consider the forces acting on the differential slice of the member as shown in Fig. 14(*b*). The total compression has been divided into two parts—the force C_1 consisting of the stress in the compressive steel and the force C_2 consisting of the concrete stress. The entire reduction factor used in computing the reduced shear V_1 is affected by the slope of C_2 with respect to the tensile steel since the force C_1 is parallel to the tensile steel. Thus, the formula for V_1 may be written

$$V_1 = V - C_2 \tan \delta_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

The formula for $\tan \delta_2$ may be written

$$\tan \delta_2 = -\frac{d}{3}\frac{dk}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68a)$$

$$\tan \delta_2 = -\frac{d}{3}\frac{dk}{de}\frac{de}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68b)$$

or

$$\tan \delta_2 = -\frac{V}{3P}\frac{d}{de}\frac{dk}{de} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68c)$$

Thus, Eqs. 67 and 68*c* appear to provide the most convenient method for calculating shear and bond stresses in terms of k rather than in terms of the section modulus. Actually, the use of the writers' formula in terms of the section modulus will require less time.

Mr. Shanley has suggested the unit shear method which has been used rather extensively in aeronautical engineering in recent years. Unfortunately, the civil engineer seldom has sufficient time in design practice to use such a method. However, the method is commonly used as a research tool to check the validity of formulas upon which the civil engineer must depend. The first of the two numerical calculations presented by Mr. Bergman illustrates the application of the unit shear method to the problem treated by the writers.

Several discussers mention the possibility of some tensile stress existing near the neutral axis. However, it is not safe to consider tensile stresses in designing horizontal members. In such members it must be assumed that a shrinkage crack exists entirely through the member at the section being analyzed. The irregular surfaces in contact at the crack can transmit shearing stress but cannot transmit tensile stress. On the other hand, it is highly improbable that vertical walls or columns have horizontal shrinkage cracks. Consequently, vertical members with large thrust eccentricity, such as cantilever retaining wall stems, are designed in a manner that is unnecessarily conservative.

Mr. Wilbur maintains that the formula $\frac{V}{bjd}$ is equally satisfactory for members with or without thrust. He presents a stress diagram in Fig. 8

which is correct for members without thrust and mistakenly assumes it to be correct for members with thrust. Hence, an incorrect value for the maximum shearing stress is computed.

Dean Grinter correctly notes that, for straight members having a length-to-depth ratio of ten to one or greater, the formula $\frac{V}{b\,j\,d}$ should give satisfactory accuracy. Actually, the eccentricity of the thrust is the criterion that determines this accuracy. It is well that Dean Grinter has called attention to the fact that, with tensile thrust, the formula $\frac{V}{b\,j\,d}$ underestimates the shear and bond stresses.

Mr. Bull offers Eq. 39*a* for computing shearing stress. This formula may be reduced to $\frac{V}{b\,j\,d}$. Hence, Mr. Bull also maintains that $\frac{V}{b\,j\,d}$ may be used for all thrust eccentricities, which is incorrect, and his stress calculations are likewise incorrect. The "line of thrust" in the writers' paper is the same as the "pressure line" in the paper to which Mr. Bull refers.[19]

Mr. Bergman has presented two instructive methods of stress calculation. These methods provide a clarification of internal stress relationships. In the second calculation Mr. Bergman introduces an unusual definition of $j\,d$ as the distance between the resultant tensile and compressive forces which remain after the thrust has been subtracted from the internal stress diagram. This definition is in strict literal agreement with the 1940 Joint Committee Code. When this definition is employed, the stress computed from $\frac{V}{b\,j\,d}$ is the maximum stress v_m at the centroid rather than the value between the neutral axis and the tensile steel. This maximum stress is not the value that the designer should compare with the allowable stress, and it is highly improbable that the writers of the code intended to recommend such a procedure.

Professor Ferguson correctly notes the artificial base upon which all reinforced concrete stress analysis rests. He presents a development which is of considerable value to the designer in showing the range of thrust eccentricity where the formula $\frac{V}{b\,j\,d}$ gives stresses that are too high with compressive thrust. Fig. 13 illustrates the matter clearly. The writers' formulas should be used in the range of small eccentricities. Structures with heavy loading, such as the structure used by the writers as an illustration, fall in this class. Most arches are in this class. Since the formula $\frac{V}{b\,j\,d}$ requires longer to evaluate correctly than the formula recommended by the writers, there is little reason to use the formula $\frac{V}{b\,j\,d}$ in any member with thrust.

Professor Ferguson has developed formulas in terms of the over-all depth h rather than in terms of an effective depth d. Beam-column stress analysis should be divided into two classifications. One class should consist of members

[19] "Computing Stresses in Reinforced Concrete Arches and Other Structures," by Anders Bull, *Concrete* (Detroit, Mich.), December, 1921, p. 242.

having the thrust within the kern and the other class should consist of members having the thrust outside the kern. In the first class the total depth h should be used in order that the thickness of the tensile steel covering will enter into the stress calculations. In the second class some conveniently defined effective depth should be used specifically to assure that the thickness of the tensile steel covering does not enter into the stress calculations.

Professor Ferguson calls attention to the fact that higher diagonal tensile stresses may exist in the compressive region, near the neutral axis, than in the region between the neutral axis and the tensile steel. In the example given by the writers this condition is very marked, because the compressive stresses in the example are very low whereas the shearing stresses are high. The high diagonal tensile stresses will exist only over a short distance along the cross section. However, they should be given some attention by future research.

Both theoretical and experimental research is needed in reinforced concrete stress analysis. Strains must be measured on actual rigid frame structures in the field. The civil engineer must learn to attach electrical strain gages to reinforcing rods as well as to insert plugs in the compressive face of members to obtain strain readings. Such field experimentation is sorely needed for the improvement of reinforced concrete design methods.

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DISCUSSIONS

ANALYSIS OF THE GENERAL TWO-DIMENSIONAL FRAMEWORK

Discussion

BY YVES NUBAR

YVES NUBAR,[20] M. AM. SOC. C. E.[20a]—Some of the comments reflect a misunderstanding—namely, that the primary object of the paper was the presentation of another method for the quick solution of special types of frames. Apparently with this thought in mind, Messrs. Polivka and Eremin recommend different procedures for such solutions. Mr. Beskin expresses a similar misunderstanding in his conclusion 5. The primary purpose of the paper, described in the "Synopsis," was an investigation of the basic properties of the general framework. Rapid solutions may be obtained as by-products, after the proper simplifications (many of them obvious) are made in the general relations to fit special cases. The writer has avoided presenting simplifications of limited application for the sake of concentrating on the general approach applicable to all cases.

Some of Mr. Beskin's comments originate from his predilection (see his conclusion 1) for "* * * special methods well adapted to each category of problems, since absolutely general methods [such as least work] are impracticable * * *." This opinion is not generally accepted. Scientific progress might be promoted best by a search for methods elastic enough to apply to the most general systems (even if processes of approximation are necessary for numerical solutions) and at the same time immediate enough to permit visualization of the geometric relationships between component elements. The method of least work meets the first requirement, but not the second. The paper has attempted to meet both requirements. Preferences are a matter of opinion, however, and it is idle to argue about them.

More of Mr. Beskin's comments seem to reflect an almost personal reaction, arising either from the fact that a few of the results which can be easily derived from the writer's general relations as special consequences have been presented

NOTE.—This paper by Yves Nubar, M. Am. Soc. C. E., was published in April, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1944, by Leon Beskin; September, 1944, by Jaroslav Polivka; and February, 1945, by A. A. Eremin.

[20] Director of Research and Development, Hub Industries, Inc., Long Island City, N. Y.

[20a] Received by the Secretary March 29, 1945.

directly by others under one name or another (mostly in connection with the analysis of continuous beams) or simply from the fact that sometimes Mr. Beskin himself can bring a solution of limited application to one of the isolated phases of the problem. This attitude permeates his discussion. The fact is (and Mr. Beskin chooses to ignore it) that an integrated and coherent method of analysis, applicable to very general frameworks, has been presented in this paper, based on concepts which are new, to the best of the knowledge of the writer and of all those connected with the preparation and examination of the paper.

Mr. Beskin derives Eq. 60 from a simple consideration of Fig. 13; it is indeed equivalent to Eq. 47, established in a more roundabout way. Mr. Beskin uses this fact as an argument purporting to show the "useless complication" of the method. However, his approach is quite specially applicable to the frame of Fig. 18; for a more complex frame he would find it impossible to proceed by such simple analysis. On the other hand, in this case as in every other one treated in the paper, a formal approach has been adopted consistently in order to conform to the routine of the general procedure applicable to all cases.

Mr. Beskin advances Eqs. 50 and 51 as arguments for denying the originality of the writer's compatibility relations, Eqs. 4, and their consequences, Eqs. 7 and 8. Mr. Beskin's equations can indeed be found in many textbooks; but, far from being the equivalents of Eqs. 7 and 8, they represent only very special (and easily derivable) consequences of these compatibility relations applied to curved beams and to triangular frames, respectively. As a matter of fact, that is precisely the way Mr. Beskin establishes his Eq. 50. He then refers to the Williot-Mohr construction, which, as he correctly implies, also has been known for years; but this again is one more particular result of these same compatibility relations (Eqs. 7 and 8) applied to articulated frames. Example 5 of the paper shows clearly the analytical translation of this construction as an application of the method. Eqs. 4, 7, and 8 are general and they are original; by his comments, Mr. Beskin unwittingly furnishes additional proof that they are also fertile.

Mr. Beskin analyzes the frame of Fig. 16(*c*) quite correctly; it is indeed of the 42d order and a rather complex structure. As adverse criticism of the method, he advances the fact that this frame requires twenty-eight rotations to be considered in the general relations. In reply, the writer would state that, comparatively, the conventional slope deflection method requires sixty-four unknown rotations, plus seven story deflections, and that, with the method of least work, the analyst must deal with some 170 unknowns. The writer had no reason for preferring the use of "arbitrary unknowns which are the same in all cases," beyond the very simple reason that the number of unknowns is appreciably less here than that in other methods. Mr. Beskin's own example is proof. Naturally, for the actual numerical solution of frames of this character, any experienced engineer will resort to processes of approximation, whether he uses one of the many existing methods, or the writer's method.

Mr. Beskin's opinions of the section under the heading, "Topology of Frameworks," are based on the fact that he can establish Eq. 52 directly, instead of

deriving it, as in Proposition (*c*) of the paper, from a general analysis. It is necessary to proceed with such an analysis in order to establish, among other important relationships, the n_i-equations required by this method for the elimination of dependent axial strains. In addition, a study of the topological properties of two-dimensional frameworks will be found a helpful steppingstone when a general analysis of space structures is attempted.

As regards Mr. Beskin's conclusions 3 and 4, the writer agrees that the unknown joint rotations that help to "open up" multiconnected frames may properly be included in the set of axial strains used by the writer as principal unknowns. However, in conformity with the general procedure of the method, these joint rotations themselves may be written in terms of the axial strains of the imaginary diagonals of some of the component parts of a frame, such as shown in Figs. 16(*e*) and 16(*f*). Therefore, the writer's introductory description of his method and the process of operations remain valid.

The method presented in this paper deals with the two-dimensional frame in its general, almost symbolized form; hence, a penalty must be paid (see Mr. Beskin's conclusion 5) in the form of more deliberate approaches for the inclusion of complex features which seldom appear in everyday frames. Mr. Beskin's own simple derivation of Eq. 60 for use in the structure of Fig. 13 proves that the method can be simplified quickly for the treatment of special cases. His several references to the excellent paper[9] by D. M. Wilson, M. Am. Soc. C. E., are irrelevant, since the writer's paper and Professor Wilson's paper have different aims.

The semigraphical method of characteristic (fixed) points described by Mr. Polivka has merit; the writer referred to an identical method in the concluding paragraph of the section under the heading, "Determination of End Moments." This method has been developed at great length as one of the steps in a previous analysis.[4] In this paper, an analytical, instead of semigraphical, approach was preferred as being simpler to present. The method described by Mr. Eremin is based on similar concepts. Either Mr. Polivka's or Mr. Eremin's method will lead to quick solutions in the case of simple frames.

[9] "Analysis of Rigid Frames by Superposition," by David M. Wilson, *Proceedings*, Am. Soc. C. E., February, 1944, p. 129.

[4] "Displacements and Stresses in the General Two-Dimensional Framework," by Yves Nubar; submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy, Faculty of Pure Science, Columbia Univ., New York, N. Y., 1941. On file for reference in Engineering Societies Library, New York, N. Y.

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DISCUSSIONS

EVAPORATION FROM A FREE WATER SURFACE

Discussion

BY A. A. KALINSKE, AND ADOLPH F. MEYER

A. A. KALINSKE,[8] ASSOC. M. AM. SOC. C. E.[8a]—Evaporation from water surfaces, and also from land areas, is an important hydrological problem on which fundamental investigative work is very scarce indeed. The author reviews, quite completely, past theoretical and experimental studies, and presents some laboratory data of his own which he correlates by use of dimensionless parameters—a method that has been quite successful in analyzing heat-transfer data. Since the analogy between heat transfer and vapor transfer in gases is very apt, the author is undoubtedly on safe ground in using this analogy in his studies.

Although mentioned by the author in connection with his discussion of the influence of pan diameter on evaporation rate, a fact that many engineers sometimes overlook is that, for evaporation from a finite area into a moving air stream, the rate of evaporation varies with down-wind distance from the windward edge. Certain theoretical analyses have been made by H. Jeffreys (55),[8b] M. A. Giblett (56), O. G. Sutton (57), and F. Graham Millar (12) which indicate how the rate of evaporation varies with down-wind distance; however, the mathematical formulations are somewhat complex and not very convenient for practical use. Since this particular phenomenon is of considerable importance in connection with measuring and predicting evaporation from various areas, it might be of interest to present briefly the results of the theoretical analyses showing at least the approximate variation of evaporation with down-wind distance. The writer does not recall any such analyses in engineering literature. The analyses are based on the turbulence-diffusion theory and several simplifying assumptions must, of necessity, be made.

NOTE.—This paper by G. H. Hickox was published in October, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by Carl Rohwer; March, 1945, by C. W. Thornthwaite; and April, 1945, by Harry F. Blaney, and Arthur A. Young.

[8] Associate Prof., Univ. of Iowa, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[8a] Received by the Secretary March 26, 1945.

[8b] Numerals in parentheses, thus: (55), refer to corresponding items in the Bibliography (see Appendix I of the paper), and at the end of discussion in this issue.

For the steady, two-dimensional case the general differential equation for diffusion is:

$$\frac{V\,\partial c}{\partial x} = \frac{\partial}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial c}{\partial y}\right) \quad \ldots\ldots (28)$$

in which, corresponding to the notation of the paper, V is the wind speed in the x-direction (horizontal wind); D_y and D_x are the eddy diffusion coefficients in the y-direction and x-direction, respectively; and c is the vapor concentration per unit volume. The change in the value of $\partial c/\partial x$ with x is negligible compared with $\partial^2 c/\partial y^2$; so the first term on the right side of Eq. 28 will be dropped. The simplest possible solution of Eq. 28 is obtained when V and D_y are assumed independent of y—that is, when they are constant. The diffusion equation then becomes:

$$\frac{\partial c}{\partial x} = \frac{D_y}{V}\frac{\partial^2 c}{\partial y^2} \quad \ldots\ldots (29)$$

The solution of Eq. 29 is:

$$c = c_0\left(1 - \phi_e \frac{V\,y}{2\,D_y\sqrt{x}}\right) \quad \ldots\ldots (30)$$

in which c_0 = vapor concentration at $y = 0$, and

$$\phi_e = \frac{2}{\sqrt{\pi}}\int_0^s e^{-s^2}\,ds \quad \ldots\ldots (31a)$$

and

$$s = \frac{V\,y}{2\,D_y\sqrt{x}} \quad \ldots\ldots (31b)$$

The normal error function ϕ_e can be obtained by reference to statistical tables. The rate at which water vapor is transferred upward is then equal to $D_y\,(dc/dy)$, according to the diffusion theory, and therefore the rate of evaporation per unit area E at any point can be found by evaluating dc/dy at $y = 0$ from Eq. 30; thus:

$$E = c_0\sqrt{\frac{D_y\,V}{\pi\,x}} \quad \ldots\ldots (32)$$

The rate of evaporation from 0 to x from a strip Δz wide is:

$$E_x = 2\,c_0\left(\frac{D_y\,V\,x}{\pi}\right)^{0.5}\Delta z \quad \ldots\ldots (33a)$$

or, in general, over a strip L feet long:

$$E_L = 2\,c_0\left(\frac{D_y\,V}{\pi}\right)^{0.5}\int L^{0.5}\,dz \quad \ldots\ldots (33b)$$

For a circular area of radius r, Eq. 33b becomes:

$$E_r = 3.95\,c_0\,(D_y\,V\,r^3)^{0.5} \quad \ldots\ldots (33c)$$

This analysis indicates that the unit rate of evaporation varies inversely as the downward distance, x, to the 0.5 power, and directly as the square root of

the wind speed. The total rate of evaporation, over the entire area, varies as the area to the 0.75 power.

Mr. Sutton derived expressions such as Eqs. 33, taking into account the variation of V and D_y upward from the evaporating surface. He assumed a velocity distribution such that:

$$V = V_1 \left(\frac{y}{y_1} \right)^{n/(2-n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

in which V_1 is the velocity at same height y_1 and in which n can vary from 0 to 1.0, its value being somewhat dependent on the turbulence present. Mr. Sutton's expressions were quite complex. He found, nevertheless, that the total mean evaporation rate, E_L, was related to velocity and area dimensions as follows:

$$E_L \propto V_1^{(2-n)/(2+n)} L^{2/(2+n)} b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

in which b is the area width. Experiments have shown that the quantity $n/(2 - n)$, for fully developed turbulent flow, is about 1/7, from which $n = 1/4$. From Eq. 35, it is apparent that E_L varies as the down-wind linear dimension of the area to the 0.89 power, and as the air velocity to the 0.78 power. The average evaporation per unit area would then be proportional to the $-$ 0.11 power of L. Mr. Sutton states that data reported by G. W. Himus (19) seem to verify these results. Mr. Millar's derivations for finite areas indicated that the average unit rate of evaporation would be proportional to $V^{0.75}$ and $L^{-0.14}$. Since Mr. Millar's method of attacking the problem was somewhat different from that of Mr. Sutton, it is significant that there is such good agreement.

The foregoing analyses indicate that the rate of evaporation from any area into a moving air stream is dependent on the distance down wind from the windward edge of that area. This item should be kept in mind when measurements of evaporation are made at some point on an area, either with pans or by the Thornthwaite-Holzman method (6). The writer believes that, in the derivation and use of the latter method, C. W. Thornthwaite and B. Holzman did not emphasize strongly enough the fact that their equation and data did not necessarily indicate the evaporation rate over the entirety of any particular area, but rather the value of this rate at some particular point.

The theory of evaporation as based on the turbulence-diffusivity-coefficient concept has advanced to a point where there is need for some extensive and carefully controlled experiments, both in the laboratory and field—to verify directly certain assumptions made in developing this theory and to evaluate certain constants present in the equations. It was with this idea in mind that, several years ago, the Iowa Institute of Hydraulic Research, at Iowa City, Iowa, with hopes of obtaining the cooperation of the U. S. Weather Bureau, became interested in an experimental program on evaporation. The writer at that time outlined the direction to be taken by the proposed laboratory and field experiments in order that direct confirmation might be obtained as to whether or not the Thornthwaite-Holzman method of evaporation measurement actually gave values of evaporation from water surfaces. So far as the

writer is aware, no such direct confirmation exists; at least he has not come across any in the literature on evaporation. It is important that direct evidence be made available before this particular method of evaporation measurement (which can have such extensive usefulness) is used too widely and confidently for obtaining evaporation data from water and land areas. The laboratory data were to be obtained in a wind tunnel some 90 ft long with a cross section 4 ft by 2 ft. In the design of this tunnel, provision was made for varying and controlling the temperature and humidity of the approaching air, temperature of evaporating water, and air velocity. Air velocity, vapor concentration, and air temperature were to be measured at various points above the evaporating surface. Turbulence measurements were to be made with a hot-wire anemometer. Field studies were planned to check and supplement the laboratory observations. It was proposed to make these studies on Spirit Lake in Iowa, which has many desirable features for conducting such a study. Actual evaporation at various points on the lake in the wind direction was to be measured with floating pans, and data were to be compared with evaporation calculations made from wind speed and humidity determinations above the lake surface.

The experiments indicated should make it possible to determine the following items definitely: (*a*) Feasibility of using the turbulence-diffusion-theory equations for computing actual evaporation, and the accuracy to be obtained from this approach; (*b*) accuracy and limitations of the method of computing evaporation from wind speed and humidity measurements at two points above a surface; and (*c*) data necessary for making predictions of evaporation from nonexistent water areas, such as proposed reservoirs.

An experimental program like the one outlined would help to provide some very important missing links between evaporation theories and actual evaporation determinations such as are needed in hydrological studies.

Because evaporation phenomena are of interest to so many different groups of scientists, the literature on evaporation is very widely scattered. In addition to the papers previously mentioned by the author and writer, some papers by R. B. Montgomery (58), H. V. Sverdrup (59), and Mr. Holzman (60) might be added to the "Bibliography." Mr. Montgomery gives extensive data on humidity distribution above an ocean surface, and in his analyses indicates that Mr. Millar (12) made some errors in his derivations. Mr. Sverdrup discusses the so-called "Bowen ratio" and indicates that, since this ratio is only approximately correct for atmospheric conditions, meteorologists are reluctant to use it. Mr. Holzman analyzes the influence of stability in the atmosphere on evaporation, and concludes that it is significant only for cold, smooth surfaces, such as snow or ice, or for low air speeds.

ADOLPH F. MEYER,[9] M. AM. SOC. C. E.[9a]—In the "Introduction," the author states,

> "An understanding of the physical occurrences of the evaporation process would lead to observations of those elements of the problem which would make the transfer of pan records to reservoirs more reliable."

[9] Cons. Hydr. Engr., Minneapolis, Minn.

[9a] Received by the Secretary March 30, 1945.

Although the writer has read the paper with considerable care and has reread many parts of it, he has no definite recollection of emphasis being placed on what observations are needed and on how the transfer from pan records to reservoirs is to be made. Perhaps in the closing discussion this matter will be amplified.

From the practicing engineer's viewpoint, the following useful information developed by Mr. Hickox may well be further emphasized:

1. The color of the evaporation pan is important as it affects heat absorption, which is one of the elements influencing evaporation. The black evaporation pan used by the author consistently showed higher evaporation than the white pan. As would be expected, the difference was small during the winter months. On an average, the black pan showed 10% more evaporation than the white pan. All pans were buried in the ground.

2. Another important fact evidenced by the author's records is the higher evaporation from the deeper pan. The pan 2 ft deep showed about 10% greater evaporation than the pans 3 in. and 6 in. deep and about 7% greater evaporation than the pan 1 ft deep. Heat absorption by the water during the daytime and less radiation at night appear to be the causes. Is it fair to conclude, in consequence, that a Weather Bureau Standard Class A evaporation pan, if 2 ft deep, would give substantially greater evaporation when similarly exposed than the present standard Class A pan?

The writer has frequently warned against the acceptance, without reservation, of a coefficient of 0.70 for reducing Class A evaporation to reservoir evaporation. In California, Arthur A. Young, Assoc. M. Am. Soc. C. E., found that a coefficient of 0.77 or 0.78 should be applied to the records of Class A evaporation pans to reduce them to reservoir evaporation. Most of the data on which the coefficient of 0.70 is based were limited to summer evaporation. However, it is a well-known fact that the difference between pan evaporation and reservoir evaporation is less during the winter months. It is also a well-known fact that reservoir evaporation is relatively much higher in fall and lower in spring. An average coefficient is meaningless in the solution of many problems faced by the practicing engineer.

The author also calls attention to the fact, always recognized, that evaporation occurs even when air and water temperatures, some distance apart in elevation, are equal, because water vapor is lighter than the dry gases of the atmosphere and not only diffuses but sets up convection currents of its own accord.

Mr. Hickox tabulates the wind correction factors (see heading, "Correlation of Results of Other Experimenters: Effect of Wind Velocity") proposed by five different investigators. In most instances, the correction factor is to be applied to the wind velocity observed near the water level whereas the writer's factor, $1 + 0.1\ V$, refers to wind velocity measured about 25 ft to 30 ft above the ground or water surface. When that factor was first published in 1915 (61), the writer stated that the wind velocity at 25 ft or 30 ft above the

ground was two and one-half to three times the velocity at about 1 ft above the ground. When this correction is applied, the writer's factor for wind effect becomes practically identical with that used by Carl Rohwer, M. Am. Soc. C. E., and stated by Mr. Hickox in the list preceding Eq. 18, namely, "$1 + 0.268\ V$."

Studies like those of Mr. Hickox are valuable and deserve encouragement. The practicing engineer, however, is sorely in need of records of air temperatures above lakes and reservoirs and of the temperature of the water in lakes and reservoirs about 1 ft below the surface, under the diverse climatic conditions that exist even within the borders of the United States. The equations for evaporation available to the practicing engineer are better than the basic data which he is forced to use in those equations. Little is known about relative air and water temperatures in the arid regions where evaporation losses are extremely high. Available records for northern lakes indicate that those of ordinary depth have summer water temperatures in excess of air temperatures. Some scattered records for the arid southwestern states indicate that lakes and reservoirs of ordinary depth in that region (probably because of the high heat loss during the night by radiation and through high daily evaporation) reach temperatures relatively lower in comparison with air temperatures than do lakes and reservoirs in the northern states where the heat received from the sun is relatively greater during the summer months than it is in the south. Water is a good absorber of heat.

The rather comprehensive study directed by the writer and published in 1942 (62) would have been more reliable in regions where unusual climatic conditions prevail if satisfactory records of comparative air and water temperatures of such regions had been available. After the work was nearly completed and it was apparent from the scattered records available that the estimated water temperatures for stations in the arid region were a little too high, corrections were made for Yuma and Phoenix, Ariz.; El Paso, Tex.; Roswell, N. Mex.; and Fresno, Calif.; but neither time nor available staff permitted corrections at other stations somewhat similarly located at which the computed evaporation appeared to be somewhat high when compared with pan evaporation records. No data are available on the basis of which a gradual reduction from about 10% in the regions of highest evaporation to zero in about the central portion of western United States can be made. The writer again cautions, however, against the use of 70% of Class A evaporation records in estimating the evaporation from lakes and reservoirs of ordinary depth. Lakes like Lake Superior obviously must be excepted. They constitute a problem in themselves. Evaporation from Lake Superior is probably not greater than 18 in. per yr (62).

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(19) **"The Evaporation of Water in Open Pans,"** by G. W. Himus, *Transactions*, Institution of Chemical Engrs., Vol. 7 (London, 1929), p. 166.

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(56) **"Some Problems Connected with Evaporation from Large Expanses of Water,"** by M. A. Giblett, *Proceedings*, Royal Soc. of London, Vol. XCIX, Series A, 1921, pp. 472–490.

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(58) **"Observations of Vertical Humidity Distribution Above the Ocean Surface and Their Relation to Evaporation,"** by R. B. Montgomery, *Paper No. 235*, Papers in Physical Oceanography and Meteorology, Woods Hole (Mass.) Oceanographic Inst., 1940.

(59) **"On the Ratio Between Heat Conduction from the Sea Surface and Heat Used for Evaporation,"** by H. V. Sverdrup, *Annals of New York Academy of Sciences*, Vol. XLIV, Article 1, 1943, pp. 81–87.

(60) **"The Influence of Stability on Evaporation,"** by B. Holzman, *ibid.*, pp. 13–18.

(61) **"Computing Run-Off from Rainfall and Other Physical Data,"** by Adolph F. Meyer, *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 1056.

(62) **"Evaporation from Lakes and Reservoirs,"** by Adolph F. Meyer, Minnesota Resources Comm., St. Paul, June, 1942.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STABILITY AND STIFFNESS OF CELLULAR COFFERDAMS

Discussion

BY J. S. MILLER

J. S. MILLER,[12] ESQ.[12a]—Professor Terzaghi stated in his paper that the most important criterion for determining the proper width of a cellular cofferdam structure has grown out of past experience. Under the circumstances it seems proper to discuss some of the practical applications of steel sheet piling and cellular structures as employed on projects with which the writer has been familiar.

Credit for the circular cell type of cofferdam must be given to Maj.-Gen. Harley B. Ferguson, M. Am. Soc. C. E., Corps of Engineers, U. S. Army (Retired), and to the late C. S. Boardman, M. Am. Soc. C. E. General (then Captain) Ferguson used the circular steel cell method in the raising of the battleship *Maine* in Havana harbor in 1910 and 1911, and this application stimulated much interest in the use of cellular cofferdams for other projects.

There are several advantages in the circular cell type of cofferdam for dam construction. The most important advantage concerns the very common situation in which the upper arm of the cofferdam must be extended across the river against the current. The current, of course, becomes more rapid as the structure is extended and the channel becomes correspondingly constricted. Furthermore, there is always the danger of freshets; therefore, it is distinctly advantageous to fill and make each main cell safe as soon as it is in place. Another advantage of circular cells is that the connecting arcs can be omitted until all the main cells are driven, thus providing partial relief of hydrostatic head while the main cells are being installed.

The cells should be filled with sand, or preferably sand and gravel, as these materials compact almost as soon as placed, especially if they are well saturated and if the cells must resist distortion and provide the greatest frictional

NOTE.—This paper by Karl Terzaghi was published in September, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1944, by G. G. Greulich, and Raymond P. Pennoyer; February, 1945, by Dean P. Tsagaris; March, 1945, by R. T. Colburn, A. F. Hedman, and Adolph J. Ackerman; and April, 1945, by Savile Packshaw, and Martin W. Lautz.

[12] Senior Vice-Pres., Dravo Corp., Pittsburgh, Pa.

[12a] Received by the Secretary March 19, 1945.

resistance to sliding. Generally, it pays to dredge the area on which the cofferdam is to be built to within 10 ft, or better still to within 4 ft or 5 ft, of rock before driving piles. Thus, the chance of the sheets encountering boulders and injuring the interlocking joints is reduced.

Most frequently, the diaphragm-wall type of cofferdam is used in protected areas (in this case, parallel with the current and relatively close to the protected shore), whereas the circular cell type is used when fighting currents such as are encountered on dam construction. Care must be taken in placing the filling in this single connecting-wall type of cofferdam. The filling must be stepped down from cell to cell to avoid distortion of the cross-walls. After being completely filled, however, the resulting cofferdam is very satisfactory in every respect.

The decision as to whether a circular cell or single connecting-wall cellular type of coffer be used also depends on the depth and character of the material through which the steel piles are driven. The single connecting-wall coffer should not be used where the sheets are driven through boulders as the sheets may drive out of the interlock. It is difficult to repair this type of coffer because of the necessity of removing and replacing a large amount of fill. Furthermore, the damaged sheets may be in the cross-wall where the break, of course, is concealed. In the case of driving through boulders, it is preferable to use circular cells as the filling of one cell can be removed for necessary repairs without affecting the adjoining cells, and with less expense and delay.

The single connecting-wall type of coffer, however, has the advantage of stiffening the coffer against deflection, especially in cases where the sheet piles are not on rock and are driven below the excavation line. The decision as to which of these two types should be used is largely a matter of judgment. It is important that the sheets be long enough to permit driving into firm material to practical refusal.

The writer's firm has conformed very closely to a width of cofferdam approximately equal to the height. If the walls were parallel, the width should be approximately 83% of the height. However, because of the loss of width due to curvature of the cells, an over-all width has been fixed arbitrarily that is equal to their height for cellular construction. A 6-in. concrete cap is customarily placed on top of the cells, on which are mounted the whirler cranes for the permanent construction work. In spite of the concrete cap, some damage to the concrete, and a small amount of washing out of the filling material, may occur during floods, and requires careful watching and prompt replacement.

In many cases the sheet-pile interlock would probably have sufficient strength to allow full hydrostatic head to the top of the sheets on the inner side of the cofferdam. Nevertheless, the writer and his associates do not like to take this chance, and consider it very good insurance to burn holes through the webs of enough sheet piles on the inner side of the cofferdam to insure the drainage of each cell to within at least 5 ft of the bottom of the excavation. This is one point that is insisted on because the writer has known of one or two instances where a cellular cofferdam failed with river stages just about

to the top. Probably, the cell first filled with water and one or more of the interlocking joints were not sufficiently strong to withstand this hydrostatic head, with the result that the inner wall failed first; and, of course, the outer wall also failed after the rupture of the inner wall permitted the coffer filling to become displaced. The draining of the cells on the inside does not ordinarily result in an increased leakage because the outer row of sheeting should exclude the water from the cofferdam area.

The writer takes exception to the statement (see heading, "Part I. Practical Considerations: Permeability of Cellular Cofferdams and of Their Sheet-Pile Enclosures") that "As a consequence, the water percolates through the dam as if the outer row [of sheet piles] were nonexistent." Admittedly, a certain quantity of seepage escapes through a sheet-pile wall—either through the interlocking joints or under the sheets—but the leakage through the interlocking joints is practically nonexistent except in coarse material. One of the reasons for using long sheeting is to prevent water from descending on the outside, passing under and coming up on the inside of a sheet-pile wall, and thus tending to loosen the material into which the sheet piles are driven, resulting in danger of settlement or loss of bearing power on the inner sheet-pile wall.

The author has made the further statement (see heading, "Part III. Conclusions") that "The assumption that the weep holes in the inner wall of cells filled with sand suffice to drain the cells is not necessarily justified." Wherever possible, in the writer's experience, sand and gravel are used for filling cellular cofferdams. As the cofferdam is unwatered, holes are burned in the inner row of a certain number of sheet piles in each cell and the progress of dewatering the cofferdam is limited by the progress of draining the individual cells. When the cells do not drain fast enough, the dewatering of the cofferdam is retarded.

It is easy to determine whether a cell is draining because, if the cell is not draining properly, moisture will appear at the joints in the sheet pile; and, if this moisture line is too high, additional holes need to be burned in the cell or the previously placed drain holes have become clogged. The important thing is to eliminate hydrostatic pressure against the inner row of sheet piles.

Although it is obvious that the water in the fill material on the back side of the cells is doubtless standing at a higher elevation than the water on the inner side, a certain length of time is required to lower the water table in the cells; hence, the cofferdam should be unwatered slowly enough to prevent undue hydrostatic head. In the design of cellular cofferdams, the writer and his associates usually provide a safe resistance against bursting of the cell for full hydrostatic head to an elevation one third of the height from the bottom to which the cofferdam will be unwatered.

The author's reference to boils on the inner side of cofferdams is most emphatically commended. Unless they come from a deep porous stratum, these boils can usually be greatly reduced by the use of long sheet piles and careful drainage of the inside walls of the cellular structure. Drainage is exceedingly

important for the simple reason that it usually tends to reduce the hydrostatic head on any boils that may develop on the inside of the cofferdam.

To prevent boils inside the cofferdam, it is sometimes advisable to place a berm or blanket on the outside of the cofferdam; thus water cannot descend and pass under the outer row of sheet piles. Fundamentally, the outer row of sheet piles is the line of defense at which water should be stopped and held. The inner wall of the cofferdam should be considered as the stability side of the structure, and for this reason the same length of piles should be used on the inside as on the outside. In case of trouble due to boils, a berm should be provided on the outside because it usually results in reducing or eliminating the boils.

From many tests of the strength of interlock of the various types of steel piling the writer has found that sections M-107, M-108, M-112, and M-113 of the Carnegie piling, and their counterparts which are manufactured under other designations by Bethlehem, Inland, Weirton, and other steel companies, are the sections best adapted for cellular construction. In general, the strength of these sections lies in the interlocking joint; and, as a general rule, a closely fitting interlock is stronger than an interlock with wide grooves and narrow tongues. In addition, one can predict the strength of an interlock almost exactly by measuring the difference between the widths of the tongues and the widths of the grooves of the individual sheets involved. Obviously, the stronger the interlocked joint is, the better (even if the sheet pile may fail in the web rather than in the interlock), for the very simple reason that the interlock is liable to be injured during driving; and the heavier the interlocked joint is made, the less likelihood there is of damage during driving.

Another danger is the occurrence of too much carbon in the steel. There should never be more than 0.35% carbon, and preferably the carbon content should be just as close to 0.30% as possible, except that, in the case of sections that have to be bent, the carbon content should be very little more than 0.25%. In one case, three cells of a cofferdam split open from top to bottom right through the web of the sheet due to the fact that there was too much carbon in the steel, and large ice cakes pounding over the top of the cofferdam caused failure.

On a few occasions, trouble has been experienced due to lamination in the web, and even in the steel-pile interlock, in spite of careful inspection both at the mill and at the site of the work. Such failures, although quite disconcerting, are very rare and do not warrant condemnation of the use of steel piling in cellular construction. Extreme care should always be taken, however, with the interlock to insure that it is properly rolled and that there are no laminated sheets.

As yet, much is to be learned about the uses that can be made of steel piling. There is ample room for improvement in the design, and also in manufacture and installation, of cellular steel-pile structures. Progress in this field since 1925 has been most gratifying and there is every reason to believe that it will continue at a rapid pace.

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DISCUSSIONS

STRESSES IN THE LININGS OF SHIELD-DRIVEN TUNNELS

Discussion

BY JACOB FELD

JACOB FELD,[23] M. AM. SOC. C. E.[23a]—A complete mathematical analysis of stresses in tunnel sections, based upon assumed loadings and resulting deformations of a circular ring, has been developed in this paper. The method of attack and the resulting tabular data provide useful tools for a rigid solution of the design problem. However, the value of the results is dependent upon the validity of the assumptions as to the soil materials and as to the relationship between acting loads or resisting reactions and deformations of the ring.

All the forces acting on a shield-driven tunnel can be divided into two classes: (*a*) Those acting in planes normal to the axis of the tunnel in the form of radial forces and tangential forces; and (*b*) those acting parallel to the longitudinal axis. It must be kept in mind that the problem is a space, or three-dimensional, problem and that the longitudinal forces will affect the radial forces and vice versa.

The science of tunnel design is dependent upon the art of tunnel construction. It is a serious mistake to design a tunnel without considering the feasibility of construction and also the conditions under which construction can proceed safely.

In an indeterminate structure of the type described in this paper, the distribution of stresses is dependent on the strain or shape of the ring. If the construction of the ring is progressing in a perfectly fluid or liquid medium, the reactions are determinate. Even in this medium, the shape of the ring does not remain circular, although the balancing effect of the air pressure tends to eliminate the required vertical resultants acting outside the ring. If it were possible to vary the amount of air pressure at various heights within the ring, a perfect balance could be obtained and the only deformation would be a radial squeezing of the ring.

NOTE.—This paper by Anders Bull, M. Am. Soc. C. E., was published in November, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1945, by J. A. Van den Broek; and February, 1945, by Nathan D. Brodkin, M. A. Drucker, and Sigvald Johannesson.

[23] Cons. Engr., New York, N. Y.

[23a] Received by the Secretary March 14, 1945.

If the tunnel is being constructed in a solid material, of sufficient internal resistance strength to permit immediate transfer of loads beyond the limits of the excavation, there is no reaction on the ring, and, of course, in this case no ring is necessary. It should be remembered that, in some solid materials, tunnels bored without shields and without rings do encounter trouble from the lateral, and sometimes also from the upward, squeeze of the rock or earth into the previously excavated tunnel section.

The case considered by the author is that of a tunnel in a saturated soil material. In this case the author makes the basic assumptions that the active pressure acting at any point in the ring is independent of the amount of ring deformation, but that the magnitude of the soil reaction at any point in the ring is directly proportional to the amount of the outward deformation from the theoretical circular shape. Information on soil reactions does not warrant such an assumption. There is no doubt that there can be no soil reaction without a strain. The value of the active pressure at any point is dependent upon the deflection at that point. This phenomenon was described a number of times by the late James C. Meem, M. Am. Soc. C. E., and termed by him "arching action over the tunnel." Mr. Meem stressed that the effects of the compressed air would be (1) to dry the soil locally above the shield, and (2) to form localized arches of rigid soil which would relieve the vertical pressure. The distribution of active pressure depends upon the change in shape of the upper portion of the circular ring. In general, those portions which deflect least will carry the greater part of the total load; and, conversely, those portions which deflect most will carry less than their share of the total load.

The amount of soil reaction at any point depends upon the amount of displacement into the soil at that point. It is questionable whether the relationship is linear. Of course, even if the relationship is linear for small deflections, the materials encountered in shield-driven tunnels are usually quite plastic, and a redistribution of soil reactions will occur in time. Just what the actual conditions will be at any one time and just how long the conditions will remain static are items of necessary information, which are lacking.

However, it seems necessary to design a ring for the worst condition of each series in the various stages of construction. When the shield is shoved forward, there must be an annular ring of soil mixed with water and air surrounding the entire perimeter which is substantially fluid. Just how long the material will remain fluid depends upon the nature of the soil in which the tunnel is built. The less clay the material contains, the sooner the liquid state will disappear. However, if the soil has a sufficient clay content to form a colloidal gel as a result of the shield compression and movement, a liquid contact ring may exist for a considerable length of time. The tunnel design must take such a possible state into account.

Farther back from the shield, the soil will be in more intimate contact with the ring. If the soil has sufficient solid characteristics, the change in the shape of the top portion may relieve the center section of the expected load and will then transfer practically all the active pressures to the sides of the tunnel ring, and possibly a large portion also to the adjacent soil. This is the condition termed "arching" by Mr. Meem.

As the shield is pushed forward, with partial removal of the soil in the front of the shield, the shear strength of the material penetrated is overcome by compression against the compressed rings. If these rings are in intimate contact with the outside soil, a portion of such force is transformed into the soil in the form of a longitudinal compressive strain. Such a strain will disappear into the soil farther away from the ring. This action shows as a relative tendency of the ring sections to separate, as if the longitudinal length increased or expanded. The result is the formation of hair cracks or separations in the tunnel lining, having the appearance usually resulting from the shrinkage of the lining.

When some methods are developed for measuring the distribution of radial forces and tangential forces during the various stages of construction, as well as for measuring the actual values of such acting forces, the necessary data will be available for a proper use of the author's design procedure. Until such information is available, it seems very doubtful to the writer that a radical decrease in tunnel lining weight and thickness can be, or should be, proposed.

Correction for *Transactions:* In February, 1945, *Proceedings*, page 246, line 3 following Table 17, after "$K = 12$" insert: "A lateral displacement of 0.51 in. at joint E (Fig. 4) due to active loads was applied to Mr. Drucker's formulas for the passive pressure and the resulting moments were determined utilizing the author's B-constants from Table 5."

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DISCUSSIONS

ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES USING REDUCED EQUATIONS

Discussion

BY I. OESTERBLOM, AND LEON BESKIN

I. OESTERBLOM,[8] M. AM. SOC. C. E.[8a]—It is interesting to note that even the ancient Greek mathematicians have something to add to modern calculus, since the basic formula, Eq. 1, of the paper was set up and used by them in discussions of "proportionality." The development of the idea to assist modern engineers, when infinitesimal differences have to be computed, no doubt will have a much greater scope than Dean Johnson has so modestly shown.

Influence lines for horizontal reactions in arch bridges have been developed before, but never so easily and gracefully. On this account, one may say that the procedure presented in the paper is too late and no longer needed. This is not so, however; there is always room for a more graceful method. Then, there are also the broader needs—better understanding of a type, clearer comparisons of different arches or of loadings (especially unusual loadings) for various designs, comprehension of the stress behavior near the danger limits for unusual loads and for comparative alternates, and knowledge of how to use materials to the best advantage. Engineers can afford to spend more time on all these questions, and they will build better balanced as well as safer structures by doing so.

For the writer, Dean Johnson's method is so far only a dream—but a fine and fair one. He hopes, however, that opportunities will come soon to turn the vision into structural beauty. For the young engineer the paper presents more than a vision; it is a challenge to build always with grace—not only bridges but all kinds of indeterminate structures.

Indeterminate structures often involve small increments, and the method should be applicable to such problems. The writer expressed similar thoughts when Hardy Cross, M. Am. Soc. C. E., first presented his great vision of

NOTE.—This paper by Lee H. Johnson, Jr., was published in November, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by Josef Sorkin.

[8] Engr., Carbide & Carbon Chemical Corp., South Charleston, W. Va.

[8a] Received by the Secretary March 28, 1945.

frame relaxation.[6] How well that idea was justified is now clear. The writer hopes sincerely that his present forecast will be equally justified.

LEON BESKIN,[9] ASSOC. M. AM. SOC. C. E.[9a]—The transformation of crude and well-known rules of the thumb used in preliminary design into a method of analysis for indeterminate structures has been attempted in this paper. It appears to the writer that this effort is misleading, because the examples selected and the internal forces computed are so chosen that they lead to erroneous general conclusions. For the special problem (hinged arches) for which the method has been developed, the charts and tables already available give enough information for preliminary design purposes without resorting to the author's approximations. Among such tables are those prepared by Eduard Winkler and presented in a French manufacturer's catalog and handbook.[10] These tables are computed for two-hinged arches, for built-in arches, and for various ratios of rise to span as well as for differences in moment of inertia along the span. Maximum positive and negative moments are given, as are the corresponding vertical and horizontal components of the force applied at the centroid, under the action of a uniform live load. The fraction of the span along which the live load is distributed is chosen to obtain the maximum moment. Tables of this type have considerable interest for a designer, and the author could have served his stated purpose better by completing existing tables of influence lines for arches of various proportions.

Theoretical Considerations.—The fact that "the basic principle is one of simple arithmetic" (see heading, "Basic Principle") reveals one limitation of the author's development: The designer must assume that factors of the type of $M\,y$ are invariably plus or minus—never both. Otherwise, it is obvious that the value of a fraction such as that on the left-hand side of Eq. 1 (in which a_1 and a_2 have opposite signs) is not represented, to any degree of accuracy, by the fraction on the right-hand side. The two fractions may have opposite signs. Thus, the value of the redundant defined by the first fraction cannot be found by application of the author's method.

It may appear that the method is acceptable when ratios such as a_n/b_n (see Eq. 3) are approximately equal, so that the error G—which the author designates as "difference term"—is small. In this case, however, the method may lead to an even more serious failure than when the error G is great, as may be observed by comparing concentrated loads (large values of G) and distributed loads (small values of G). This fact can be demonstrated theoretically as follows: The values of the thrust computed by the correct formula (H) and by the approximate formula (H'), respectively, are

$$H = \frac{\Sigma M\,y\,\Delta s/I}{\Sigma y^2\,\Delta s/I} \quad \ldots\ldots (33a)$$

and

$$H' = \frac{\Sigma M\,y\,\Delta s}{\Sigma y^2\,\Delta s} \quad \ldots\ldots (33b)$$

[6] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[9] Design Specialist, Consolidated Vultee Aircraft Corp., San Diego, Calif.

[9a] Received by the Secretary March 30, 1945.

[10] *Aciéries de Longwy*, Longwy Steel Co., Paris, France, 1928 Ed.

Introduce the moments M' corresponding to H':

$$M' = M - H' y \quad \ldots\ldots (34)$$

which can be considered as statical moments. Introduced into Eqs. 33, the thrust correction is found to be equal to zero if the approximate formula (Eq. 33*b*) is used, or to G if the correct formula (Eq. 33*a*) is used; thus:

$$G = \frac{\Sigma M' y \, \Delta s/I}{\Sigma y^2 \, \Delta s/I} = \frac{\Sigma M y \, \Delta s/I}{\Sigma y^2 \, \Delta s/I} - \frac{\Sigma M y \, \Delta s}{\Sigma y^2 \, \Delta s} \times \frac{\Sigma y^2 \, \Delta s/I}{\Sigma y^2 \, \Delta s/I} = H - H' \quad \ldots (35)$$

The final moment M_0 can be expressed in terms of M' by the relation:

$$M_0 = M - H y = M' - G y \quad \ldots\ldots (36)$$

Since M and y are assumed approximately proportional to each other, the quantities M' are very small in comparison with the quantities M. Thus, $G y$ and M' are of the same order or magnitude, and the error resulting from the use of H' (leading to M') instead of H (leading to M_0) may yield considerable differences between the moments, although the thrusts are practically equal.

Application to a Two-Hinged Arch.—The significance of the foregoing comments can be applied to the paper under discussion by the analysis of a two-hinged parabolic arch, under two conditions of loading. The following comparison is made between an arch that has a constant value of $E\,I\,dx/ds$ and an arch defined by the formula,

$$\frac{1}{E\,I} = \frac{1}{E\,I_0} \frac{ds}{dx} (1 + \beta \, \xi^2) \quad \ldots\ldots (37)$$

in which $\xi = \frac{x}{l}$, the distance x being measured from the center line. The constant β may be positive or negative. Eq. 37 defines the arch ribs of types 3 and 6 (Fig. 5(*b*)). The extreme values of m and m' are + 40 and − 0.975 (see Tables 3 and 4), which correspond approximately to $\beta = -0.975$ and $\beta = 40$, since the relation between m (or m') and β is

$$(1 + m)(1 + \beta) = 1 \quad \ldots\ldots (38)$$

Case (*a*). Two Symmetrically Placed Concentrated Vertical Loads (P).—The expression for the thrust is (see Fig. 4, with $\alpha = 1 - a/l$):

$$\frac{H}{P} = \frac{l \int_0^\alpha (1 - \alpha)(1 - \xi^2) \, ds/I + l \int_\alpha^1 (1 - \xi)(1 - \xi^2) \, ds/I}{h \int_0^1 (1 - \xi^2)^2 \, ds/I} \quad \ldots (39a)$$

or

$$\frac{H h}{l P} = \frac{\int_0^\alpha (1 - \alpha)(1 - \xi^2)(1 + \beta \, \xi^2) \, d\xi + \int_\alpha^1 (1 - \xi)(1 - \xi^2)(1 + \beta \, \xi^2) d\xi}{\int_\alpha^1 (1 - \xi^2)^2 (1 + \beta \, \xi^2) \, d\xi} \quad \ldots (39b)$$

The final value is:

$$\frac{H h}{l P} = \frac{5(5 - 6\alpha^2 + \alpha^4) + \beta(3 - 5\alpha^4 + 2\alpha^6)}{32(1 + \beta/7)} \quad \ldots\ldots (40)$$

Table 8 contains the coefficients for thrust and moment, resulting from the application of Eq. 40, for various values of α and β. Moments are determined at the points of application of the concentrated loads. Percentage errors in thrust and moment, as related to the "basic arch" ($\beta = 0$), are included for comparison.

TABLE 8.—INFLUENCE VALUES RESULTING FROM EQS. 40 AND 43

Constant β	Case (a)								Case (b)	
	Thrust, $\frac{hH}{lP}$				Moment, $\frac{M}{lP}$				Moment, $\frac{M}{l^2 w_2}$	
	Computation		Error (%)		Computation		Error (%)		Computation	Error (%)
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.5$		
− 0.975	0.802	0.551	− 3	−1	0.196	0.101	+12	+12	−0.00934	+27
− 0.900	0.800	0.550	− 3	−1	0.200	0.102	+10	+11	−0.00956	+24
0	0.781	0.556	0	0	0.219	0.113	0	0	−0.0119	0
+10	0.708	0.580	+10	+4	0.292	0.160	−25	−29	−0.0212	−44
+40	0.676	0.590	+15	+6	0.325	0.179	−32	−37	−0.0254	−53

In his presentation, Dean Johnson has not shown the errors in moment values, which reach 37%. He has reduced the error in thrust (which reaches 15%) to a fictitious value of 6% (see line 14 preceding Eq. 26) by the use of average arches with varying moments of inertia that have no relation to the basic arches introduced at the beginning of the paper. Since the object of the paper is to establish a method based on the properties of the basic geometric arches, it seems inconsistent to use arches with varying moments of inertia for the evaluation of errors due to the use of the basic arch. The result is an apparent compromise between the actual arch and the basic arch, which the writer cannot accept as tenable.

Case (*b*). A Variable Distributed Load (w).—Consider a variable load defined by

$$w = w_0 + w_2 \xi^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

When the ratio w_2/w_0 is small, the load is approximately uniform; and, when the ratio is large, the variation in load is relatively great. Then the formula for thrust is

$$\frac{hH}{l^2} = \frac{1}{2} w_0 + w_2 \frac{\int_0^1 (1 - \xi^2)\,(1 = \beta\,\xi^2)\,(1 - \xi^4) d\xi}{12 \int_0^1 (1 - \xi^2)^2\,(1 + \beta\,\xi^2)\, d\xi}$$

$$= \frac{1}{2} w_0 + w_2 \frac{6 + \beta}{9\,(7 + \beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

and the moment at the crown is:

$$\frac{M}{l^2} = \frac{w_2}{12}\left[\, 1 - \frac{4\,(6 + \beta)}{3\,(7 + \beta)} \,\right] = -\frac{(3 + \beta)\, w_2}{36\,(7 + \beta)} \ldots\ldots\ldots\ldots (43)$$

Values of the moment and of the error resulting from the use of the basic arch for this condition are included in Table 8. They show that errors are greater in the case of distributed loads than in the case of concentrated loads. The relative error in the value of the thrust is a function of the ratio $\rho = w_0/w_2$; thus:

$$\epsilon = \frac{\beta}{7\,[6 + \beta + 9\,\rho\,(7 + \beta)/2]} \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

The error is infinite if ρ has the value,

$$\rho = -\frac{2\,(6 + \beta)}{9\,(7 + \beta)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

Eq. 45 shows that, with a proper choice of ρ, any arbitrary value of ϵ can be selected. For instance, if $\epsilon = -2$, the approximate value of the thrust is equal and opposite to the actual value.

Loading conditions that lead to such results are not theoretical. For instance, in frames, used to reinforce a shell structure such as an airplane fuselage, it is possible to find cases for which the frame tends to open or close under a given applied load, depending on the distribution of the moments of inertia. Thus, depending on the structure, there may be thrusts of opposite signs—a fact that cannot be shown if basic arches are used.

Incidental Comment.—The author introduces expressions such as "difference term" and "reduced equation" when the expressions, "error" and "approximate equation," are well known and fulfil the same requirements, without obscuring the approximate character of the method. The expression, "correspondence of ordinates," is loosely defined. For instance, in Fig. 1(*c*), there is no reason to assume that the correspondence is stopped precisely at the center line. The concept of "correlation," which has a definite significance in statistics, could be usefully applied, because it would supply a definite quantity to express the "degree of correspondence" between various curves.

The author has devoted too much of his paper to a description of elementary methods of integration and to well-known tricks, such as the use of two symmetrical loads and the expedient of integrating along the half span when determining the thrust due to one load. The distinction between m and m', to separate positive and negative values, leads to the writing of two sets of formulas instead of one.

A difference is claimed between parabolic and circular arches, which actually does not exist. For example, in the sentence following Eq. 27, the author states: "The influence values [for circular arches] * * * are not a linear function of L/h, as in the case of the parabola." This is correct for a true "basic arch"; but, because the author uses a substitute basic arch in the case of a parabolic profile for expediency (in which $I\,dx/ds$ is constant, instead of I), he finds that the thrust is a linear function of L/h. If a similar adjustment is made for circular arches, the property of linearity may be shown to exist for this second case as well as for parabolic arches.

Conclusions.—In recent years, many writers have tried to simplify the analysis of redundant structures. With a few exceptions (which all concerned methods of successive approximations), these attempts have proved failures. This paper, likewise, has failed in its purpose.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

COLUMN FORMULAS

Discussion

BY RALPH E. SPAULDING, AND JAMES H. CISSEL

RALPH E. SPAULDING,[15] M. AM. SOC. C. E.[15a]—In the study of column formulas it is not generally recognized that the Euler formula,

$$\frac{P}{A} = \frac{K\,\pi^2\,E}{(L/i)^2} \qquad (34)$$

gives true theoretical values of P/A for all values of L/i from zero to infinity when, and only when, the yield strength is equal to the modulus of elasticity ($\sigma_y = E$). Since this relationship does not exist in metals, it is necessary to modify, or complete, the Euler formula in order to adapt it to any required use. This may be done by introducing the yield-strength factor into the Euler formula; thus:

$$\frac{P}{A} = \frac{K\,\pi^2\,E}{(L/i)^2} \times \frac{\dfrac{K\,\pi^2\,E}{(L/i)^2} + \dfrac{\sigma_y\,(L/i)^2}{K\,\pi^2} + (E + \sigma_y)}{\left[\dfrac{K\,\pi^2\,E}{(L/i)^2} \times \dfrac{E}{\sigma_y}\right] + \dfrac{\sigma_y\,(L/i)^2}{K\,\pi^2} + (E + \sigma_y)} \qquad (35)$$

Eq. 35 can be applied to any metal that follows Hooke's law by simply inserting the respective values of σ_y, E, and the end condition factor K. When σ_y becomes equal to E, Eq. 35 reverts to the Euler formula.

It is interesting to test this formula in connection with available data concerning the values of σ_y and E. In Fig. 2(*a*), the writer has plotted Eq. 35 for values of L/i as high as 400, for three metals with different characteristics. Apparently, this formula fulfils all the desirable requisites for a column formula as enumerated in this paper.

If the yield strength and the modulus of elasticity are known, the values may be inserted in Eq. 35 and the resultant values of P/A may be determined with reasonably dependable accuracy for all values of the ratio of slenderness.

NOTE.—This paper by William R. Osgood was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by L. E. Grinter, George Winter, and Jonathan Jones; and March, 1945, by Edward Godfrey.

[15] Cons. Engr., Jacksonville, Fla.

[15a] Received by the Secretary February 26, 1945.

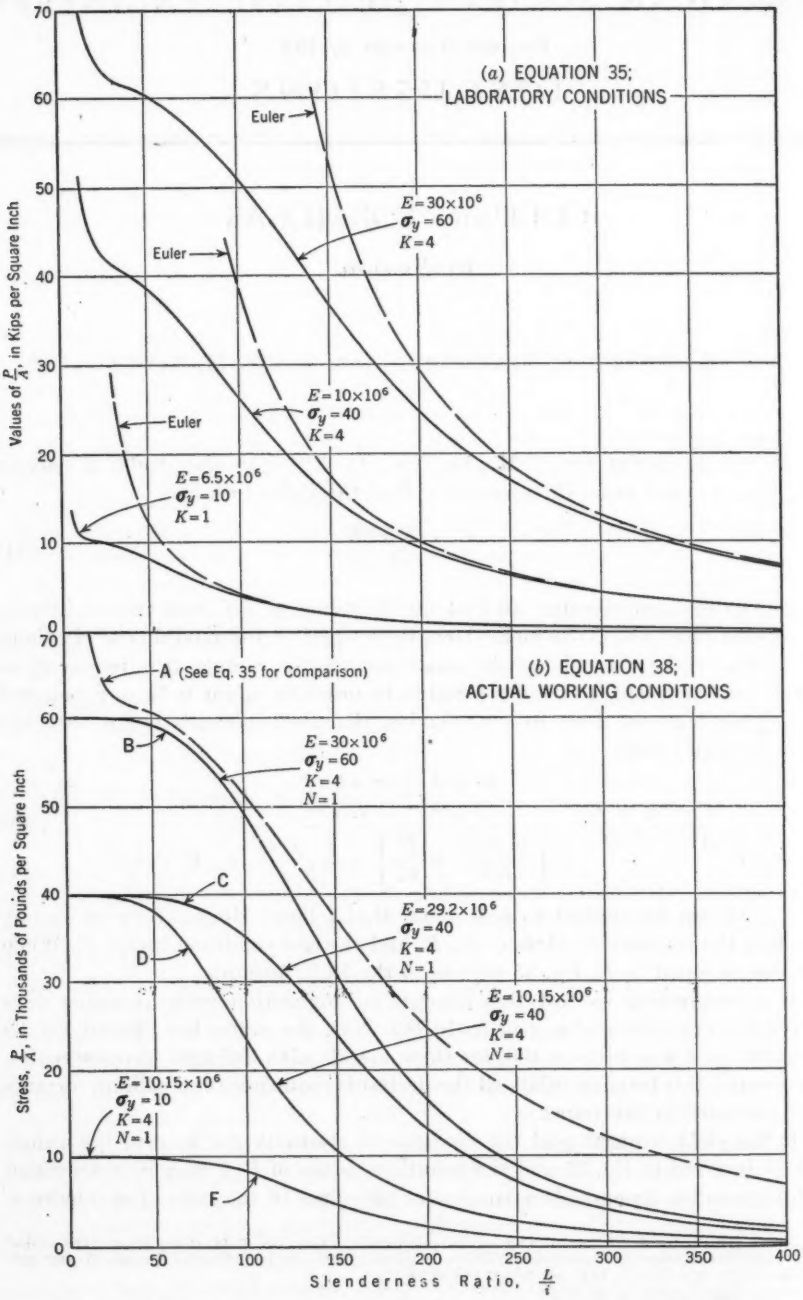

FIG. 2.—COMPARISON OF COLUMN FORMULAS FOR STEELS WITH VARIOUS PROPERTIES

There is a distinction between the column formulas proposed for the determination of allowable stresses in columns as built in engineering structures and formulas that reflect the probable ultimate load on columns at failure under laboratory conditions. Probably the author did not intend to open for discussion the relative merits of these many formulas. The column behavior under laboratory conditions can be predicted with reasonable accuracy; but, because of unexpected eccentricities and undetected defects in manufacture and erection, the design of columns for practical use becomes more a matter of judgment and experience aided by theory based on observed laboratory tests. The laboratory tests indicate the maximum limits beyond which the working formula cannot go.

Hundreds of formulas have been devised, and used, for the determination of the working stresses in columns. The writer takes the liberty of suggesting one more.

In columns, as built into structures (in contrast with columns tested under perfect laboratory conditions), the end condition factor, K, is a variable rather than a constant. For example, in a flat end column this factor would be 4 when the column was very short; but, as the column became relatively longer and the effects of manufacturing defects, residual stresses, etc., began to have more influence on the ultimate strength, the factor should be reduced. Reduction in accordance with the following formula is suggested—

$$K\text{ (effective)} = K\text{ (initial)} \times \frac{\sigma_y}{\sigma_y + (L/i)^2} \quad \ldots\ldots\ldots\ldots (36)$$

A somewhat simplified and closely approximate form of Eq. 35 with the factor of safety, N, added is—

$$\frac{P}{A} = \frac{\sigma_y}{N} \times \frac{1 + \dfrac{K\pi^2 E}{\sigma_y (L/i)^2}}{1 + \dfrac{K\pi^2 E}{\sigma_y (L/i)^2} + \dfrac{\sigma_y (L/i)^2}{K\pi^2 E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

and, by inserting the value of K proposed by Eq. 36 for the K-value in Eq. 37—

$$\frac{P}{A} = \frac{\sigma_y}{N} \times \frac{1 + \dfrac{K\pi^2 E}{\sigma_y (L/i)^2 + (L/i)^4}}{1 + \dfrac{K\pi^2 E}{\sigma_y (L/i)^2 + (L/i)^4} + \dfrac{\sigma_y (L/i)^2 + (L/i)^4}{K\pi^2 E}} \quad \ldots\ldots\ldots (38)$$

which is the writer's suggested column formula for allowable working stresses in columns as built. The value of the end condition factor, K, still remains to be inserted in accordance with the actual initial end condition. The factor of safety can best be determined by judgment of existing conditions and requirements.

Eq. 38, like Eq. 35, is applicable to any metal or alloy that follows Hooke's law and for any value of the ratio of slenderness—in contrast to many formulas that are applicable only to one metal and often only within a narrow range of L/i. Although Eq. 38 is not quite as simple as some others that do not have

such universal applicability, it is a reasonably simple formula, having only one variable together with its reciprocal.

In Fig. 2(*b*), curves are plotted from Eq. 38. Curve A (the only curve plotted from Eq. 35) shows the probable ultimate column strength under laboratory conditions for an alloy with the properties indicated in Fig. 2(*b*), whereas curve B for the same metal, and for actual working conditions, gives the suggested maximum design stress before application of a desired factor of safety. Curves C, D, and F (Fig. 2(*b*)) are of the same type as curve B.

Curves C and D illustrate the comparison in column strength between two metals having the same yield strength but very different moduli of elasticity; and curves D and F illustrate the difference between two metals having the same modulus of elasticity but quite different yield strengths. When the ratio of slenderness is from about 120 to 140, column C has an allowable working stress equal to twice that of column D although the respective yield strengths of the columns are the same. Column D has an allowable working stress only twice that of column F, whereas the yield strength of metal D is four times that of metal F. This relationship may be verified by reference to actual test data. In other words, as has always been known, except for very short columns, a high elastic modulus is of greater economic importance than a high yield strength.

JAMES H. CISSEL,[16] M. AM. SOC. C. E.[16a]—In the "Synopsis," the author states that

> "The * * * requisites of any column formula * * * are that the average stress be a continuous, monotonically decreasing function of the ratio of slenderness and that it approach the Euler value as the ratio of slenderness becomes infinite."

It may be questioned whether this requirement in itself is an objective of formulas for compression members. The main purpose of any column formula should be to express, in convenient form, the behavior of a column in terms of dimensions and critical load.

The "Summary" states that "There is little to be said for a formula on the grounds of its simplicity * * *." This viewpoint completely ignores the need of the practicing engineer for a procedure of estimating the capacity of a given member quickly and accurately.

Any acceptable column formula must correctly appraise the probable ultimate axial load on a column by proper consideration of any combination of the following four possible modes of failure: (1) Compressive failure by crushing the material; (2) column failure or bending over the unsupported length of the member (in this type of action, the cross sections are translated laterally in the direction of least resistance of the member); (3) torsional failure in which the cross sections are rotated with respect to the longitudinal axis of the member; and (4) plate buckling or local failure of elements of the section by wrinkling over a limited length of member.

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[16] Prof., Structural Eng., Asst. to Director of Eng. Research, Univ. of Michigan, Ann Arbor, Mich.

[16a] Received by the Secretary March 29, 1945.

Failure (1) is the type that would occur in a very short member (l/i less than, say, 20) with elements or walls of such ratios of width to thickness as are necessary to preclude local wrinkles or buckles. Current specifications such as those of the American Railway Engineering Association (A.R.E.A.) or the American Institute of Steel Construction prescribe minimum thickness and maximum values of width to thickness for plates and elements; and, hence, short members complying with such specifications will not fail until the yield stress in the material has been developed by an axial load.

Failure (2) implies bending over the length of the column induced by eccentricity of loading and crookedness of member. If the thickness of elements is sufficient to prevent local wrinkling, failure will not occur until the stress on the extreme fiber due to direct stress and bending has reached the yield stress of the material. The appraisal of this action is the general purpose of column formulas, and the unstated assumption incorporated in the commonly used equation is that the elements of the section are elastically stable for unit stresses up to the yield stress of the material.

Failure (3) represents a twisting of the section with respect to the longitudinal axis, and is particularly likely to occur in members lacking torsional resistance, or subject to eccentric loads. It does not appear to have great practical importance for members with the proportions usually found in average structures, but may become significant for unsymmetrical sections or for long, slender members.

Failure (4) is sometimes called local failure by buckling of elements comprising the section. Current specifications attempt to eliminate this type of failure from consideration by specifying minimum thicknesses and proportions of parts, such as outstanding legs of flange angles. Such limitations merely prevent the use of thin-walled members, which may, in fact, be perfectly satisfactory and economical for a given purpose.

As a result of its extensive study of steel columns, the Society's Special Committee on Steel Column Research[9] recommends the secant formula as best interpreting the test results. Allowing for adaptation to the author's notation, this formula is commonly expressed in the form:

$$\sigma_w = \frac{P}{A\,\eta} = \frac{\dfrac{\sigma_y}{\eta}}{1 + a \sec \dfrac{l'}{2\,i}\sqrt{\dfrac{P\,\eta}{A\,E}}} \qquad (39)$$

in which σ_w is the allowable working stress; η is the safety factor equal to unity for ultimate load; P is the ultimate axial load; σ_y is the yield strength of material; A is the cross-sectional area; a is equal to $e\,c/i^2$ equal to the eccentric ratio; i is the radius of gyration of section; l' is the free length between inflection points; and E is the modulus of elasticity.

The validity of the secant formula has been proved repeatedly by test, and probably is as rational an equation as can be developed for expressing the ultimate axial load on columns that fail in accordance with failures (1)

[9] *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1376.

and (2). It cannot be safely applied in the form expressed in Eq. 39 to thin-walled sections or to members that are critical as regards torsional resistance. Moreover, since it is difficult in practical cases to determine precisely the value of the eccentric ratio expressed by the term a in Eq. 39, the use of a is inconvenient for practical design purposes. In recognizing the latter objection, the Society's committee recommended[9] the use of an average value of 0.25 for a, and a parabolic equation for values of l'/i to 160. In following this recommendation, the A.R.E.A. permits the use of the following formula for allowable working stress on structural grade steel columns:

$$\sigma_w = 15{,}000 - 0.33\left(\frac{l}{i}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Eq. 40 gives a close approximation of values obtained from Eq. 39, when $\sigma_y = 33{,}000$, $a = 0.25$, $l' = 7/8\,l$, and $\eta = 1.76$. It should be observed that Eq. 40 will produce results consistent with those obtained by the secant formula, only when $a = 0.25$. This value, set by the Society's committee after a comprehensive study of the tests, provides an allowance for eccentricity of loading and eccentricity due to crookedness of member, as follows:

$$\text{Eccentricity of applied loading} = 0.15\,i^2/c$$
$$\text{Eccentricity due to crookedness} = 0.10\,i^2/c$$

Thus, for a cross section whose radius of gyration is $0.8\,c$, the load is assumed to be applied at a distance of $0.096\,c$ from the axis, and the deviation of the axis from an absolute straight line is assumed to be $0.064\,c$, regardless of the length of the member. Also, the allowance for crookedness was based on a tolerance of $\frac{1}{1{,}200}\,l$ for a member with length $l = 100\,i$, and with a radius of gyration $i = 0.8\,c$. The effect of the eccentric ratio in producing variation in values of P/A is shown diagrammatically in Fig. 3.

Considering the fact that, in nearly all actual cases, the allowances for eccentricity in the secant formula are mere guesses, the writer prefers the Johnson formula as a means of estimating column capacity. This formula expresses the behavior as a parabolic curve tangent to the Euler curve when $P/A = 1/2\,\sigma_y$. Thus:

$$\sigma_w = \frac{P}{A\,\eta} = \frac{\sigma_y}{\eta} - \frac{\frac{\sigma_y^2}{\eta}}{4\,C\,\pi^2\,E}\left(\frac{l}{i}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which l is the unbraced length of member and C is a factor determined by the conditions of end restraint affecting the member (1.0 for hinged ends and 4.0 for fixed ends).

When $\frac{P}{A} = \frac{1}{2}\sigma_y$, $\frac{l}{i} = 4.44\sqrt{\frac{C\,E}{\sigma_y}}$; and, for greater values of $\frac{l}{i}$, the Euler formula is used, in which

$$\frac{P}{A} = \frac{C\,\pi^2\,E}{(l/i)^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)$$

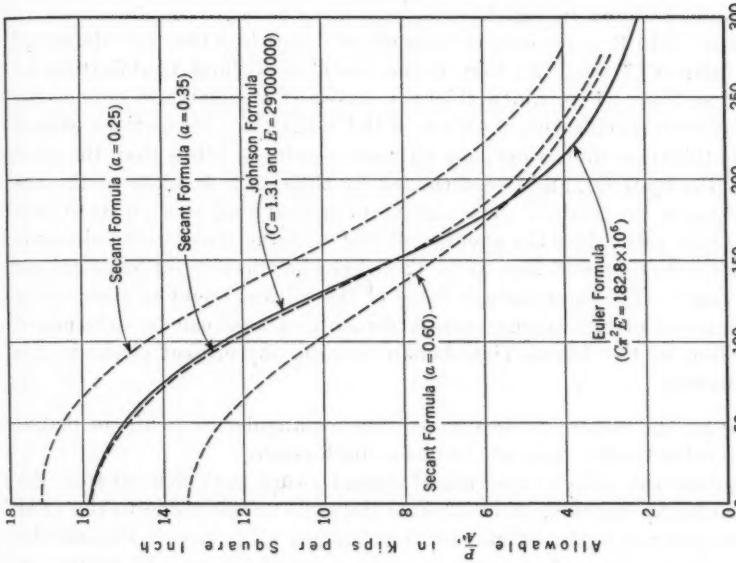

FIG. 4.—COMPARISON OF SECANT FORMULA, EQ. 39 ($\eta = 1.55$ AND $\sigma_y = 33$ KIPS PER SQ IN.), WITH JOHNSON FORMULA, EQ. 41 ($\eta = 2.09$ AND $\sigma_y = 33$ KIPS PER SQ. IN.)

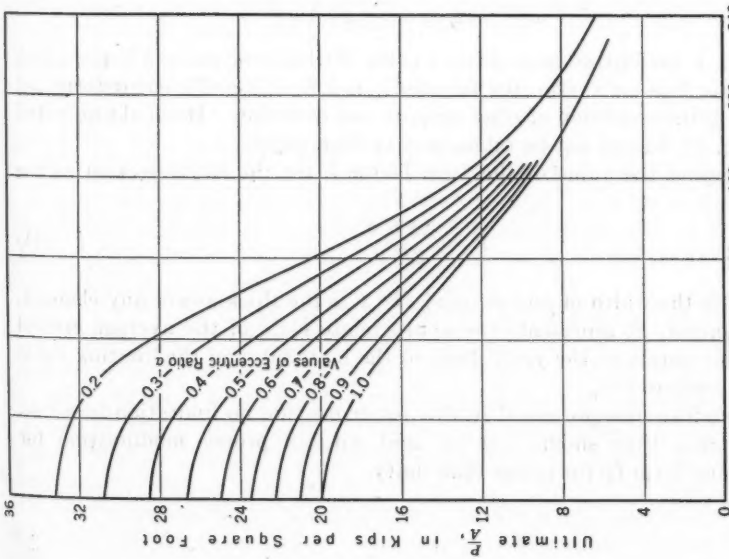

FIG. 3.—ULTIMATE AXIAL LOADS BY THE SECANT FORMULA, EQ. 39 ($\sigma_y = 40$ KIPS PER SQ IN., $\eta = 1$, $l' = \frac{1}{2}l$ FOR PIN ENDS, AND $E = 29.4 \times 10^6$)

A comparison of the Johnson-Euler curves with those obtained from the secant formula is shown in Fig. 4.

Since Eqs. 39 to 42 apply only to cases where failure is of the types described herein as failures (1) and (2), they cannot be used without modification for thin-walled sections where width:thickness ratios of elements are outside the limits set by such specifications as those of the A.R.E.A. For sections outside such width:thickness limitations, the ultimate resistance is less than the yield strength of the material, and is determined by the elastic stability of the elements composing the section. The maximum critical load on such sections is that load which will induce the average critical buckling stress in the elements comprising the section, and Eqs. 39 to 42 will not be effective for loads greater than this value. The approximate value of the critical buckling stress for a section composed of rectangular-shaped elements of area can be determined by application of the Bryan-Timoshenko formula for critical stress in the following manner:

(*a*) Assume the sections to be divided into rectangular elements, or plates, attached to other similar elements at one or both edges.

(*b*) Compute the value of the critical stress for each such element from the Bryan-Timoshenko formula, and designate the ratio of this value to the yield stress of the material as the *z*-factor for the element. The Bryan-Timoshenko formula for the local crippling stress in rectangular plates may be written in the form:

$$z = \frac{\sigma_{cr}}{\sigma_y} = \frac{K\,\pi^2\,E}{12\,\sigma_y\,(1-\mu^2)}\left(\frac{t}{b}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

in which σ_{cr} is the critical unit stress; t is the thickness of plate; b is the width of plate; μ is Poisson's ratio (0.3 for steel); and K is a coefficient determined primarily by the condition of edge support and restraint. It should be noted that, in Eq. 43, z must not be taken greater than unity.

(*c*) Compute the value of the form factor F for the entire section as the ratio:

$$F = \frac{\Sigma k\,b'\,t'}{\Sigma b'\,t'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

in which b' is the width of any element and t' is the thickness of any element. The form factor, F, represents the approximate ratio of the average critical stress on the section to the yield stress of the material, and the limiting value of P/A is therefore $F\,\sigma_y$.

Since the formulas presented in this paper provide no limitation based on crippling stress, they should not be used without proper modification for sections whose form factor is less than unity.

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DISCUSSIONS

UTILIZATION OF GROUND-WATER STORAGE IN STREAM SYSTEM DEVELOPMENT

Discussion

BY C. W. SOPP, AND J. A. BRADLEY

C. W. SOPP,[6] M. AM. SOC. C. E.[6a]—Ground water is of vast importance in the economic development of the midwestern and western states. That ground-water storage is an important factor for preserving water resources is recognized by artificial recharge in certain localities and by limitation of the pumping draft to the safe yield in other areas.

The author mentions the recharge of ground-water basins by diverting water to wells which are not being used. This practice generally has not been successful because of reduction in the intake capacity of the wells. The wells act as a sand filter in reverse; and, if they are not equipped with pumps, there is no opportunity for backwashing. Bacterial growth in the wells and in the surrounding alluvium may also reduce the rate of infiltration. It might be necessary to filter and chlorinate water to be infiltrated into basins by means of wells. Near Lytle Creek, in California, a shaft was dug to a depth of 250 ft, all of which was through gravel except 8 ft. Openings were left in the cribbing throughout the length of the shaft. Initially, the shaft had an infiltration capacity of from 5 cu ft per sec to 6 cu ft per sec, but after seven years this capacity had been reduced to 2 cu ft per sec.

The total storage capacity of the ground-water unit may not be available for economic use for the following three reasons: (*a*) The "specific yield" or available porosity decreases with depth due to consolidation of the alluvium; (*b*) where storage is developed by pumping, only the storage within the range of pumping lifts is usable; and (*c*) all storage should not be exhausted, thus making the annual supply in the ground-water unit fluctuate in accordance with its annual recharge.

In one basin studied by the writer the "specific yield" decreased from 15% to 7% at 150 ft greater depth. Mr. Conkling lists many factors that may

NOTE.—This paper by Harold Conkling was published in January, 1945, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1945, by Raymond A. Hill; and April, 1945, by Clarence S. Jarvis, and John R. Charles.

[6] Asst. Chf. Engr. and Gen. Mgr., Pasadena Water Dept., Pasadena, Calif.

[6a] Received by the Secretary April 2, 1945.

enter the hydraulic equation to determine the safe yield of a ground-water unit. The safe yield equals the water entering the unit minus water losses, including leakage. Records of the requisite data may not exist, and the data are often difficult to estimate. The safe yield, being the difference between fairly large quantities, is subject to large error if this method of computation is followed.

Mr. Conkling also gives a method of determining the safe yield by using the factors of draft and change in storage but mentions the difficulty of evaluating change in storage due to the obscure factor of "specific yield." Nevertheless, this method has the advantage of making many records or estimates unnecessary as it gives the safe yield from all sources without the necessity of segregation. Satisfactory computation of the safe yield can be made by this method in many cases.

In the use of ground-water storage, the true occurrence of ground water as a subterranean stream, or as in a basin, is of great importance under the laws of certain states.

The underflow of a river may follow the course of the surface channel; but part of the underflow may leave the surface channel at some point and may not rejoin it. The interrelation between a surface stream and its underflow is more evident when the stream is traversing a mountain valley, where it may be in contact with or support the surface flow, than after the stream has debouched upon a broad plain. In this latter case, part of the underflow may follow an ancient, buried channel of the stream, and water moving in such an ancient underground channel may conform to the classification of ground water flowing in a defined channel. Other ground water flowing in the general region and perhaps in the same direction may not, strictly, be traversing a definite underground channel and may more properly conform to the definition of percolating water. Water flowing in an ancient channel cannot be developed independently of other overlying ground water.

The San Gabriel River and basin cited by the author illustrate certain differences in occurrence of ground water in a subterranean stream with a known and definite channel and basin water. In the canyon the underflow is confined between the impervious canyon walls and is clearly a subterranean stream with a known and definite channel. Below the canyon mouth in San Gabriel basin, but upstream from the Paso de Bartolo (Whittier Narrows), the ground water is far below the stream bed. The boundaries of the ground-water body are somewhat indefinite and are several miles apart—in fact, the width is about eight tenths of the length. The direction of movement of ground water in the basin generally follows that of the surface stream, but the ground water in the basin clearly occurs as percolating water. Immediately above the Paso de Bartolo the movement is again like that of a subterranean stream (see Fig. 3). D. G. Thompson (48)[6b] states that he

> "* * * does not know whether any Court decision has classified the movement of ground-water in the San Gabriel Basin as 'percolation,' but he believes that it is a proper classification according to the customary con-

[6b] Numerals in parentheses, thus: (48), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

ception of that term as it is commonly used except perhaps for minor parts of the basin where certain geological conditions produce special conditions * * *."

Beneath the water table in a basin there may be distributary arteries of principal movement and of water in a buried channel, but the whole may be properly classified as percolating water.

The flow of water through the alluvium in a ground-water stream or through a basin as percolation is caused and controlled by the same hydraulic laws that govern percolation through a porous medium. It should be well recognized that ground water in a basin is never at rest but moves from points of recharge to points of extraction or discharge. The courts have merely applied a name, "percolating water," to classify ground water moving through a basin to distinguish it from ground water moving in a channel. The author seeks to remove the mystery of movement of ground water from the minds of laymen, and some courts by stating (see heading, "Characteristics of Alluvial Valleys") that "* * * it is moving as a stream or group of streams through the unit * * *." Such a mystery does not exist in the minds of hydrologists and this comparison may cause legal confusion. The comparison and any extension to classify all ground waters as streams lead to legal difficulties in certain states depending on the various state laws.

The rate of movement of ground water is important if a large portion of the recharge to a basin is localized as percolation from a stream overlying the basin. Ground-water movement is usually a combination of displacement and the actual movement of a particle of water. A study of water-table fluctuation and of chemical constituents of water in wells in the Raymond basin indicates that a displacement wave travels at the rate of about 124 ft per day, whereas the actual movement of a particle of water was only 8 ft per day.

Storage, rather than flow, is the dominant feature in the occurrence of ground water, whereas, for a stream, flow is the principal feature. The subflow of canyons is illustrative of the small amount of water flowing in ground-water streams. In San Gabriel canyon, the Edison Company's diversion dam developed 50 miner's inches of subflow; and the subflow at the Morris Dam of the City of Pasadena was 20 miner's inches. The subflow at the dams of the Los Angeles County Flood Control District on other streams was measured during construction as follows:

Dam	Miner's inches
Pacoima	2.3
Sawpit	5.3
Dalton	1.4
Tujunga	12.0

Contrasting with the flow of ground-water streams is storage in basins such as the Raymond basin. Here the specific yield is about 2,000 acre-ft per foot of depth; and, assuming 100 ft of fluctuation of the water table, the total usable storage is 200,000 acre-ft, whereas the annual movement in and through the basin is about 31,000 acre-ft. This large storage capacity equates the supply from wet and dry periods and has supported an overdraft since about 1917–1918.

In nature a ground-water basin is in equilibrium—that is, the water entering the basin is discharged as a regulated flow at the outlet or by evaporation and transpiration in areas where the water table is reached by the root zone of vegetation. In the natural state there is no safe yield; the outflow equals the inflow. Disturbance of natural conditions by pumping from the ground water creates a safe yield by reducing the outflow and facilitating percolation from overlying rainfall and streams. The safe yield of a basin can be increased by ground-water drafts, limited by economical lifts, up to the point where the outflow ceases, thereby utilizing the complete water supply of the hydrologic system, provided that the basin storage capacity is adequate. Development of the safe yield of a basin is accompanied by increased pumping lifts; but, with the present art of pumping and return from crops, water can be produced from lifts of several hundred feet with resulting profit to growers of crops. In utilizing the storage capacity of a ground-water basin, the water table will fluctuate over wet and dry periods. A greater fluctuation is more feasible where the pumping lift is small than where it is large.

In certain states ground water must be divided into the following classes: (*a*) Water flowing in defined subterranean streams; and (*b*) percolating water.

To judge the water storage capacity in alluvium, it is necessary to distinguish between these two classes of ground water. The reasons for such distinction are largely legal, pertaining to water rights. Distinction is unnecessary in certain states, such as Utah, Idaho, and Nevada, where permits are required for well drilling. Section 42 of the Water Commission Act of California provides that:

> "* * * whenever the terms 'streams,' 'stream system,' 'lake' or 'other body of water,' or 'water' occur in sections of this act relating to applications to appropriate water or permits issued pursuant to such applications * * * such terms shall be interpreted to refer only to surface water and subterranean streams flowing through known and definite channels."

C. S. Kinney (49) states:

> "* * * that the word 'defined' means a contracted and bounded channel, though the course of the stream may be undefined by human knowledge; and the word 'known' refers to the knowledge of the course of the stream by reasonable inference."

The presumption that ground waters are percolating runs through many court decisions. The burden of proof is upon the party who asserts that a defined underground stream exists. The rules of law in certain states pertaining to surface or subterranean streams, flowing in known and definite channels and percolating ground water in a basin, differ as follows:

A. Application of the Water Commission Act of California would require permits from the State Division of Water Resources for drilling wells to ground-water streams, whereas no permits are required for wells drawing percolating water from a ground-water basin.

B. Owners of wells drawing from a ground-water stream can acquire rights adverse to downstream producers but the contrary is not true. This rule is

not applicable to percolating water in a basin where all pumping is mutually adverse.

C. Under section 20*a* of the California Water Commission Act, 3 years of nonuse of water from a ground-water stream would be interpreted as constituting a forfeiture of the right, whereas under court decisions 5 years of nonuse of water from a basin is required for a forfeiture.

The maximum supply from a stream system requires use of the large storage capacity of the underflow and of any ground-water basin to which the stream is contributory. Such storage requires no dam and is not subject to evaporation losses although the leakage may be large.

J. A. BRADLEY,[7] ASSOC. M. AM. SOC. C. E.[7a]—A comprehensive and excellent discussion of ground-water reservoirs on certain stream systems in the western United States and of the determination of their utilization from an engineer's standpoint is presented by Mr. Conkling. It is common knowledge that man's need for supplemental water in most of the alluvium ground-water provinces shown in Fig. 1 is increasing. No better evidence can be found than the already constructed works and further contemplated developments for the control of flood waters and the prevention of uneconomic "evapotranspiration" losses from areas of high ground water which frequently occur at the lower ends of ground-water basins. These flood and wasted waters must be controlled and conserved if additional supplies are to be obtained. As Mr. Conkling has stated (see heading, "Characteristics of Alluvial Valleys"), full utilization of the water supply of the stream systems of the west cannot be attained without balancing the advantages and disadvantages of surface and underground storage, which will ultimately result in the use of "some part of the storage possibilities of the unconsolidated alluvium."

The terms, "ground-water reservoir" and "ground-water basin," are generally used synonymously. As Mr. Conkling explains, the term, "basin," is used very loosely; but it is a convenient term, nevertheless. However, the term, "ground-water basin," may be considered univocal. A surface stream has a bed and banks; a lake has a bottom and shores; but a basin is a depressed land form and has a "bottom" and a marginal boundary or "rim." Geologically, basins may have resulted from the operation of subterranean forces or may have originated in the constructional or destructional processes of nature (50). If the term, "basin," is applied to a ground-water basin, the "bottom" is the foundation upon which a more porous valley fill material rests—generally considered the basement complex underlying that porous material. The "rim" may be an elevated extension of the bottom; but, in any event, it is a physiographic or geologic feature acting like an obstruction to the movement of ground water. The material composing the bottom and rim will have at least one or more characteristics to distinguish it from the porous material which it surrounds or at least partly surrounds (the rim may

[7] Asst. Flood Control Engr., Orange County Flood Control Dist., Santa Ana, Calif.

[7a] Received by the Secretary April 13, 1945.

be nonexistent along parts of an "open" type basin). The degree of imperviousness of bottom and rim is, or may be, only a relative matter. However, the term, "reservoir," indicates an extra supply or storage detained for future usefulness and, in the writer's opinion, should be preferred in describing the performance of a ground-water body.

According to the writer's conception of essential requirements, a ground-water reservoir in alluvium should have the following characteristics:

(*a*) Water in the porous material everywhere in contact and moving in compliance with the physical laws of seepage and percolation;
(*b*) Capacity sufficient to yield water of good quality, in commercial quantities, by pumping at feasible cost;
(*c*) Capability of being recharged either naturally or artificially; and
(*d*) Some escape as outflow (surface or subsurface or both), to prevent the water from becoming so saline as to be unsuitable for use.

Since the introduction of the deep-well turbine pump, the development of ground-water supplies in Southern California basins has increased rapidly. The water table in these various ground-water reservoirs has shown considerable annual, as well as cyclic, fluctuation. One inference that might be gained from Mr. Conkling's paper is that, once a ground-water reservoir reaches a stage of development in which the water table is lowered, the supply may actually be increased because of greater percolation opportunity and improved recharge conditions caused by change from native to cultivated vegetation. In consequence, more emphasis has been placed recently on studies of methods for replenishment by ground-water investigators.

As the author has indicated, a ground-water basin may consist of two parts —a pressure area (containing confined water), and a forebay or intake area (containing a free water table). However, changes in storage are possible only in the intake area. Where this combination occurs, it appears to be an inherent characteristic of an underground basin in alluvium that the intake area is around the higher boundaries of the basin. Only a small portion of the intake area is generally used by nature at normal flood times for distributary channels to carry runoff. In intermittent streams, crossing alluvium water is lost underground from the beds of the distributary channels: First, by the physical laws of seepage; and second, after field capacity has been reached, by those laws attendant upon percolation. In any event, the movement of water is predominantly downward to the water table (unless intercepted by impervious strata and thence carried laterally and downward to the water table). Naturally, an attempt to increase the wetted area of these channels was made; hence, off-stream spreading was resorted to where topographically feasible on adjacent porous lands having sufficiently pervious soil types.

Spreading water for storage in underground reservoirs has already been described by A. T. Mitchelson and Dean C. Muckel (5). However, the writer wishes to add to the experience of recharging ground-water reservoirs through the use of wells drilled solely for that purpose by illustrating the performance (see Fig. 9) of a typical well drilled in the quaternary alluvium of the Orange

County, California, coastal plain. The well is one of six, driven in a peripherally-leveed spreading ground of 70 acres, constructed and operated by the Orange County Flood Control District for the purpose of sinking water underground. This spreading ground overlies a part of the intake area of one of the ground-water reservoirs of the South Coastal Basin of California (27).

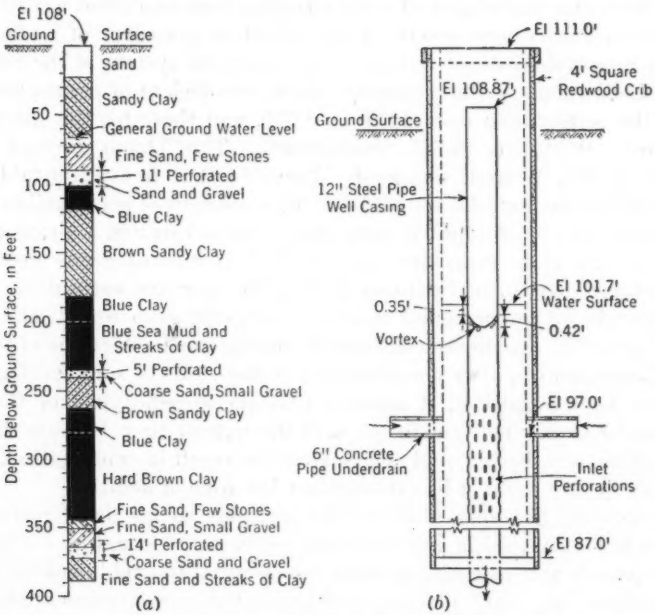

FIG. 9.—GRAPHIC LOG AND OPERATION OF TYPICAL RECHARGING WELL

The supply to the spreading grounds consists wholly of the storm-water runoff collected from an entire city area (which formerly had no definite outlet channels) by a system of storm drains. It is delivered by gravity to the spreading area, where it is permitted to flood over the entire surface. The water is absorbed and cleared of silt by the porous sandy soil, and then drains into a system of 6-in. concrete tile drains laid at an average depth of 10 ft below the surface in a rock-filled trench. These drains empty into each side of 4-ft square redwood intake cribs constructed around each well. The rotatory motion of the water is induced as it passes through the intake perforations in the well tube to the underground strata.

In discussing the performance of a ground-water reservoir, Mr. Conkling has justly stressed the necessity for determining the specific yield of the water-bearing material—an important factor in estimating storage capacity. The writer believes that, in those underground reservoirs of considerable extent where pumps are drawing from confined water (and it is feasible to salvage wasted waters from areas of high ground water), knowledge of the permeability of the underlying aquifers is equally desirable if the long-time average safe yield is to be estimated and the ground-water reservoir is to be

managed efficiently. Transmissibility is generally determined as the coefficient of permeability and, as stated by Leland K. Wenzel, Assoc. M. Am. Soc. C. E. (51), and C. F. Tolman (8) but as credited to O. E. Meinzer, is defined as the rate of flow in gallons a day through a square foot of cross section of material, under a hydraulic gradient of 100%, at a temperature of 60° F. In 1940 the writer had charge of some pumping tests on drilled wells for determining permeability and specific yield (effective porosity) of water-bearing materials in a ground-water reservoir on the stream system of the Santa Ana River in California. Values computed for the coefficient of permeability from some of the tests varied from 4,880 to 6,070, and those for the specific yield varied from 16.3% to 18.7%, respectively. The Thiem method (51), as described by Mr. Wenzel, was used. Provided that free water-table conditions prevail in the material tested, both the coefficient of permeability and the specific yield may be computed from data obtained during a single pumping test. Although it is expensive, the method is advantageous because the results obtained are for undisturbed material lying in the natural state.

Well-developed underground reservoirs operated on a sound management basis will generally experience considerable fluctuation in elevation of the water table. Consequently, after consideration of the views of some geologists and pedologists (soil scientists), it appears that the specific yield of the water-bearing material may not remain constant throughout time, because the effects of compaction, weathering, and chemical action result in cementation. These effects may extend more or less throughout the zone of aeration.

Mr. Conkling has ably outlined the methods so far developed and the objectives to be attained in ground-water reservoir investigation. Continuity of proper records and wise management are essential to avoid depleted ground-water supplies if the many planned works for future construction are to accomplish their purpose.

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(8) **"Ground Water,"** by C. F. Tolman, 1st Ed., McGraw-Hill Book Co., Inc., New York, N. Y., 1937. (*a*) p. 469.

(27) **"South Coastal Basin Investigation; Geology and Ground Water Storage Capacity of Valley Fill,"** by Rollin Eckes, *Bulletin No. 45*, Div. of Water Resources, Dept. of Public Works, State of California, Sacramento, Calif., 1934.

(48) *Transactions*, Am. Soc. C. E., Vol. 102 (1937), p. 806.

(49) **"Treatise on the Law of Irrigation and Water Rights,"** by C. S. Kinney, 2d Ed., Vol. II, Sections 1154 and 1155.

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(51) **"The Thiem Method for Determining Permeability of Water-Bearing Materials and Its Application to the Determination of Specific Yield,"** by Leland K. Wenzel, *Water-Supply Paper No. 679-A*, U. S. Geological Survey, U. S. Government Printing Office, Washington, D. C., 1936.

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DISCUSSIONS

INTERRELATION OF CERTAIN STRUCTURAL CONCEPTS

Discussion

BY RALPH W. STEWART, AND R. C. BRUMFIELD

RALPH W. STEWART,[7] M. AM. SOC. C. E.[7a]—The flexure of a continuous structure exhibits a number of interrelated phenomena both in the structure as a whole and in the individual spans. These phenomena comprise what are known as the "beam constants" and the "load constants," as well as a variety of other constants or properties that can be derived from the beam constants and the load constants. Each of the various methods for computing the bending moments in a continuous structure is contrived by selecting a limited number of these properties and using the properties chosen in a more or less regimented manner. A conversion table showing the interrelationship between the constants used in a number of systems of analysis has been presented by Fang-Yin Tsai.[8] The table covers the general case—namely, members with variable cross sections.

The final test of a structural concept is its usefulness in computing the stresses in a structure. The author's concept features nodal distances, restraint factors, and final carry-over factors. These terms may be applied to the solution of a continuous structure such as that shown in Table 2, which gives the computations for the relative moments at end E of the frame analyzed. For any moment applied at point E, 2/347 of the moment will be transmitted to end A, 3/347 of it to end G, etc. This frame was selected because a very thorough exposition of its analysis by the concept of end moment distribution is available for reference.[2] Fig. 4 shows the additional work required to determine the effect at all joints of an applied moment at point D. Restraint factors are computed as follows:

$$r_{DC} = \frac{6 \times 0.7666}{3 \times 0.7666 - 1} \times 5 = 17.7$$

NOTE.—This paper by Camillo Weiss was published in January, 1945, *Proceedings*.

[7] Bridge Engr., City of Los Angeles, Los Angeles, Calif.

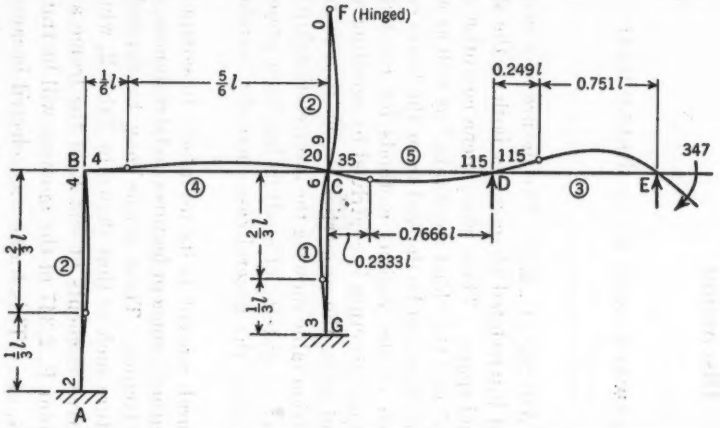
[7a] Received by the Secretary March 9, 1945.

[8] *Transactions*, Am. Soc. C. E., Vol. 102 (1936), Table 3, p. 44.

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

TABLE 2.—Illustrative Example

(Successive Steps Shown by Numerals in Parentheses)



Line	Concept	Member AB	Member BC	Member CD	Member DE	Member CF	Member CG
1	Nodal intercept	(1) (Known) 1/3	(3) (Eq. 9) 5/6	(5) (Eq. 9) 0.7666	(7) (Eq. 9) 0.751	(9) (Known) 0	(10) (Known) 1/3
2	Carry-over factor	(2) (Known) 1/2	(4) $\frac{1/6}{5/6} = \frac{1}{5}$	(6) $\frac{0.2333}{0.7666} = 0.304$	(8) $\frac{0.249}{0.751} = 0.332$	(Not needed)	(Not needed)
3	Restraint factor	(Not needed)	(11) (Eq. 10) 13.33	(Not needed)	(Not needed)	(12) (Eq. 10) 6	(13) (Eq. 10) 4
4	Relative moments	(14) AB = 2 (assumed) BA = 2/0.5 = 4	(15) BC = BA = 4 CB = 4/0.20 = 20	(See line 5)	(See line 5)	(16) CF = 6/13.33 × 20 = 9	(17) CG = 4/13.33 × 20 = 6
5	Relative moments	....	....	(18) CD = 20 + 9 + 6 = 35 DC = 35/0.304 = 115	(19) DE = DC = 115 ED = 115/0.332 = 347	....	....

and

$$r_{DE} = \frac{6 \times 1}{3 \times 1 - 1} \times 3 = 9.0$$

The moment M_{DE} equals $9/17.7 \times 115 = 58.5$; and the total fixed-end moment is $58.5 + 115 = 173.5$. Fig. 4 then indicates that 2/173.5 of a moment applied at point D is transferred to end A, etc.

The solutions of Eq. 9 to obtain the nodal intercepts for members CD and DE require an appreciable length of time. Also, the nodal distances in this structure and other continuous structures are different for each member and are different for the ends of the same member. This makes the nodal distances inferior to the moment-area constants as bases for analyzing a structure.

Fig. 5 shows a solution which uses the moment-area constants delineated in the form of an elastic curve traverse. All angles of flexure are at the one-third points of the span. They do not move in response to changes in restraint as do the nodal distances. Thus, any labor or difficulties in computing the locations of the angles are eliminated.

An angle of flexure multiplied by the stiffness of the member involved gives a moment, and a moment divided by the stiffness of the member gives the appurtenant flexure angle. By using these properties of Fig. 5 in connection with simple geometry, the arithmetical computations for Fig. 5(*a*) (which yields the same result as Table 2) can be performed in much less time than that required for the example in Table 2. This condition is also true for Fig. 5(*b*) which corresponds to Fig. 4. Fig. 5(*c*), which shows the final carry-over factor to all joints for an applied moment at point B, furnishes an automatic check of the computations for Figs. 5(*a*) and 5(*c*) by noting that Fig. 5(*a*)

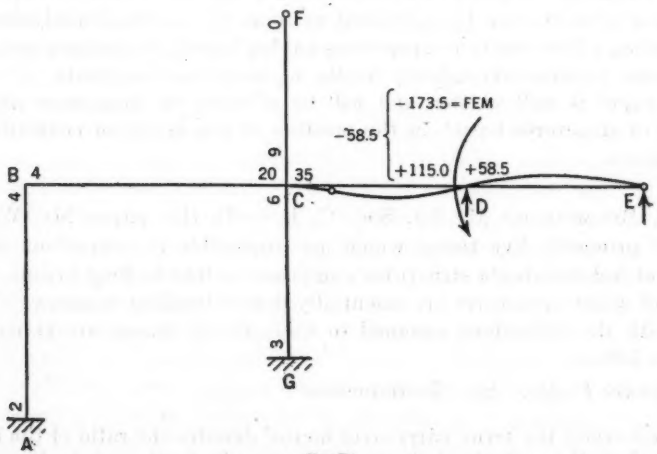


FIG. 4

gives an applied moment of 347 at end E for a joint rotation of unity at point B and that the computations for Fig. 5(*c*) for a rotation of unity at end E yield a moment of 347 at point B, as required by Maxwell's theorem of recipro-

cal deflections. The computations for Fig. 5(*d*) require only one setting of a slide rule from which two readings are taken, followed by a moment summation about joint C. Fig. 5(*d*) gives the final carry-over factors at all joints for a

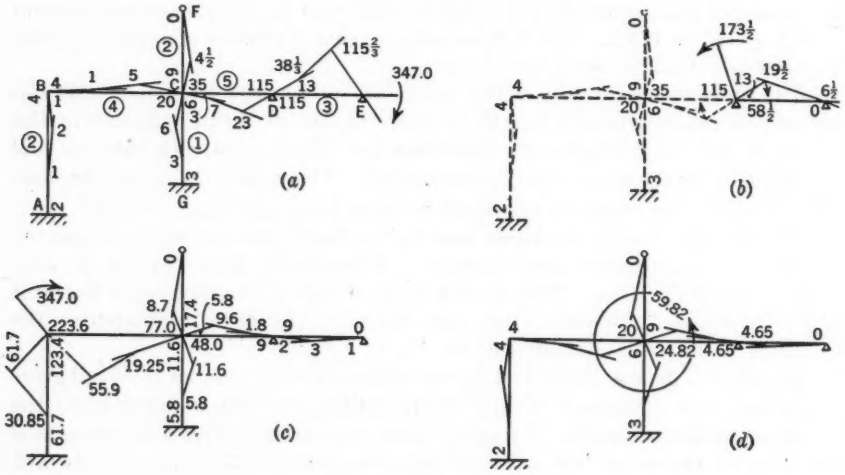


FIG. 5

moment applied at joint C. All the necessary arithmetical work is shown directly in Fig. 5. For a solution based on nodal distances the additional work represented by Table 2, and scratch-paper work not shown, are necessary.

The foregoing solutions demonstrate how the structural concepts which the author presents can be appraised as tools for practical analytical work. An extension of the study to structures having tapering members would show even greater relative adaptability for the moment-area constants.

The paper is well written and will be of value to computers who make analyses of structures based on the position of the points of contraflexure of the members.

R. C. BRUMFIELD,[9] M. AM. SOC. C. E.[9a]—In this paper Mr. Weiss has discussed primarily five terms which are applicable in connection with the solution of indeterminate structures composed of interlocking beams, the distortions of which structures are essentially due to bending moment. The five terms, with the definitions assigned to them in the paper, are given by the author as follows:

Carry-Over Factor.—See "Introduction":

"In this study the term 'carry-over factor' denotes the ratio of the moment induced at the restrained, but not necessarily fixed, end to the moment applied at the other end of a member which is on unyielding supports at both ends."

[9] Associate Prof., Civ. Eng. Dept., Cooper Union, New York, N. Y.

[9a] Received by the Secretary April 3, 1945.

(By unyielding supports it is obvious that the author means supports that allow no movement of translation although the beam may rotate.)

Stiffness or Restraint Factor.—See "Stiffness Factors and Distribution Factors":

> "* * * the resistance to rotation of the end of a member will be E times a stiffness factor or restraint factor 'r.' For a member with constant moment of inertia, r then varies between 3 K and 4 K."

Distribution Factor.—See sentence containing Eq. 13:

> "[The distribution factor] measures the ratio in which an external moment applied at the joint B will be distributed [to a particular member of the joint]"

Degree of Restraint.—

> "* * * the degree of restraint Q_{AB} will be defined as the ratio of the restraint exerted upon the end of a member to the resistance against rotation of the entire joint."

Degree of Fixity.—

> "Applying a rotation θ_B to end B of an unloaded member AB, the degree of fixity at end A is the difference between unity and the ratio of the slope at the restrained end A to the slope at A if the end were hinged."

It seems clear from the context that the author has in mind the use of these terms largely in connection with the Cross method[2] of solving indeterminate structures by distribution through successive approximations. This appears to be evident from the statement in the "Introduction" and from the fact that, in deriving an expression for the distribution factor (Eq. 13), only that one is derived for an artificial moment applied to the joint externally, this being the one required in the Cross method of distribution.

The justification for such terms as those listed must lie wholly in the relative number of cases in which they are found useful, such as (*a*) the services that they may be made to render in computing the bending effects of loads applied to an indeterminate structure; (*b*) the allocation of these effects to the separate members of the structure; and (*c*) the evaluation of the distortions induced in the structure. Of these purposes the first two will ordinarily be the more important. Concepts that serve these purposes adequately and with a maximum of simplicity should be adopted and used. Those which do not meet these tests eventually will be discarded.

As a basis for deriving his various formulas, the author uses the concept of the "fixed point," or "node point" as he terms it. This concept has long been in existence and is used to express the fact that, in any beam whose bending-moment diagram consists exclusively of the effects of the bending moment introduced at one end of a span and that induced by it at the opposite end of the same span, the point of zero bending moment remains at a fixed point along the beam regardless of the size or sense of the inducing moment.

[2] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

The selection of the term "node point" for this position of zero bending moment is unfortunate, particularly when taken in connection with the context of the paper in which the term "elastic curve" is used to identify this point. In vibratory motion, the most important characteristic of a node is that there is no movement of the vibrating body at the node. This is not true of the "fixed point" of a beam. In fact, there is always lateral movement at its fixed point, when the beam is only under the influence of a moment at one end and its induced moment at the opposite end. The casual reader is likely to form quite an opposite impression in the paper from the statement (see heading, "Fixed Points and Carry-Over Factors"): "* * * the wave-like elastic curves will pass through a certain node."

The idea of fixed point might well be eliminated entirely from the considerations, in connection with the five definitions listed at the beginning of this discussion. Although the author has used the concept as a basis for his derivations, it need not have been so used. Every equation in the paper could have been derived with equal or greater simplicity if the concept had not been introduced, using as a basis well-established ideas that have long existed. Judging the term, then, from the standpoint of service it seems to be superfluous.

The author uses the terms "stiffness factor" and "restraint factor" interchangeably as measures of the bending moment required to be applied to the end of a beam, resting on a free support, in order to rotate that end through a unit angle under the influence of the restraints that may exist at the opposite end of the same beam. Such a beam might be span AB, Fig. 2, the restraints that influence the beam being those at joint B.

The writer would suggest a change in this concept. Stiffness is characterized by movement or, rather, capacity to resist movement. The author measures this movement by the bending moment required to cause it. This has one virtue and one fault. It represents a stiff beam in terms of a large number and so appeals to the natural instinct that this should be so. Its fault lies in the fact that the stiffness is not measured in terms of movement, but rather, in an auxiliary fashion, in terms of the moment required to cause movement.

If the movement itself is used as a measure of stiffness, the term "stiffness factor" or preferably "stiffness index" may be defined as a measure of the angle of rotation caused at the end of a beam by the application there of a unit couple. Such a definition allows stiffness to be measured directly in terms of movement or rotation. The fact that it is now an inverse measure of the movement (that is, a large "stiffness index" represents a relatively flexible member) does not mitigate against the definition since by it a true concept is obtained of the movement of the beam end. Thus the word "index" is used instead of the word "factor" which ordinarily is thought of as a multiplier.

One should also distinguish sharply between the terms "stiffness factor" (or "stiffness index") and "restraint factor" (or "restraint index") which terms the author uses interchangeably. Stiffness is a function that is (and should be) influenced largely by the characteristics of the beam itself. Restraint, on the other hand, is something applied from without and beyond the

beam, which is applied only to the beam end. In such a beam as span AB, Fig. 2, the term "stiffness index" should be applied to end A, whose rotation is influenced largely by beam AB, and the term "restraint factor" (or "index") to the far end of beam B, where rotation is influenced by the three restraining beams, BJ, BC, and BH. Thus, when a moment is transmitted from point A to point B, the rotation of end A of the beam is measured by the stiffness index, and that of the same beam at end B, by its restraint index. Such a concept leads to simple expressions for the various terms under discussion.

Referring to Fig. 2, if a unit couple is applied to span AB at end A, the angle of rotation at end A is:

$$6\,E\,\theta_A = \frac{2 - f_{AB}}{K} \qquad (24)$$

From Eq. 24, one may define "stiffness factor" (or "index") by the equation:

$$i_{AB} = \frac{2 - f_{AB}}{K} \qquad (25)$$

Thus, the index is shown to be a measure of angular rotation. Without giving the derivations that lead to the conclusion, the expression for the restraint index for span AB, at B, is given by the expression:

$$R_{AB} = \frac{1}{\frac{1}{i_{BJ}} + \frac{1}{i_{BC}} + \frac{1}{i_{BH}}} \qquad (26a)$$

or

$$R_{AB} = \frac{i_{BJ}}{1 + i_{BJ}\left[1 + \frac{i_{BJ}}{i_{BC}}\left(1 + \frac{i_{BC}}{i_{BH}}\right)\right]} \qquad (26b)$$

The right-hand quantity of either Eq. 26*a* or Eq. 26*b* is usable, but Eq. 26*b* is preferable since it is adaptable to slide-rule use without any other mathematical work except mental additions. The usefulness of this restraint index is apparent from the fact that the bending moment arriving at joint B, from end A, divides to the three restraining members according to the following expressions:

$$M_{BJ} = M_{BA}\frac{R_{AB}}{i_{BJ}} \qquad (27a)$$

$$M_{BC} = M_{BA}\frac{R_{AB}}{i_{BC}} \qquad (27b)$$

and

$$M_{BH} = M_{BA}\frac{R_{AB}}{i_{BH}} \qquad (27c)$$

or, in general:

$$M_n = M\frac{R}{i_n} \qquad (28)$$

It is seen that Eq. 28 has a close relationship to the distribution factor given in Eq. 13. It differs from this factor in that it gives the division, to the

restraining members, of moment arriving at the joint through the transmitting member or actuating member AB, rather than the division of the moment to the members when an artificial torque is applied to the joint and all radiating members are acting as transmitting members.

From Eqs. 24 and 28 it is clear that, if the bending moment which arrives at joint B, through span AB, is unity, the angle of rotation of joint B induced thereby is

$$6\,E\,\theta_{BJ} = 1 \times \frac{R_{AB}}{i_{BJ}} \times i_{BJ} = R_{AB} \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Thus R_{AB} becomes a direct measure of the rotation of the end B of beam AB, just as i_{AB} was a direct measure of the rotation at the opposite end. The expression R_{AB} involves only the characteristics of the restraining members and thus becomes a true restraint index, whereas i_{AB} involves mostly the characteristics of the beam, restraint being only included through the expression f_{AB}. Thus "stiffness index" is a proper name for this term.

With these concepts in mind the author's expression for the carry-over factor, freed from its special application in the Cross method of analysis as explained in the paper, becomes:

$$f_{AB} = \frac{1}{2 + K_{AB}\,R_{AB}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

which compares with Eq. 16.

In the writer's opinion the term "degree of restraint" discussed in the paper is not especially useful. In terms of the foregoing concepts and the author's definition it would be the ratio of the restraint index of member AB to the restraint index of the joint as a whole, against the artificial torque applied to it. Thus

$$Q_{AB} = \frac{\dfrac{1}{\dfrac{1}{i_{BJ}} + \dfrac{1}{i_{BC}} + \dfrac{1}{i_{BH}}}}{\dfrac{1}{\dfrac{1}{i_{BA}} + \dfrac{1}{i_{BJ}} + \dfrac{1}{i_{BC}} + \dfrac{1}{i_{BH}}}} = 1 + \frac{R_{AB}}{i_{BA}} \quad \ldots\ldots\ldots\ldots (31)$$

The degree of fixity, using the definition of the paper, may be derived as follows, using the beam noted in the definition and assuming a unit couple applied at joint B: With the beam as restrained, Eq. 24 yields—

$$6\,E\,\theta_A = \frac{2f_{BA} - 1}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32a)$$

With end A assumed resting on a support that permits perfect freedom of rotation—

$$6\,E\,\theta'_A = \frac{2 \times 0 - 1}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32b)$$

Then:

$$F_{AB} = 1 - \left(\frac{2f_{BA} - 1}{K} \times \frac{K}{-1}\right) = 2f_{BA} \quad \ldots\ldots\ldots\ldots (33)$$

These examples serve to show that the introduction of the idea of "restraint index" for the beam as differentiated from "stiffness index" permits the mathematical delineation, in quite simple forms, of all the concepts listed in the paper. The elimination of the idea of "fixed point" (or "node point" as it is called) in no way hinders the work, and the derivations leading to the results are, in most cases, simpler than those listed in the paper.

The author's project of codifying the nomenclature used in connection with indeterminate structures is praiseworthy. It should lead to beneficial results if properly handled. Such standardization, if well done, will be an assistance in clarifying thought and mutual understanding in the engineering profession. There is one danger to be noted, however: If too rigidly applied and if there is too great insistence upon the letter of the standard rather than the spirit, the result may be that engineering thought is put into a "strait jacket" and initiative stifled. The number of concepts involved is large when indeterminate structures are to be considered in their broadest sense. A noncoplanar frame, involving not only beams of variable section but also curved beams, is a case in point. In such a structure, the number of interrelated factors is sufficient to exhaust the letters in the ordinary alphabet as well as those in the Greek alphabet. Every one in a position to influence standardization should be patient and move cautiously until the literature on these matters is much more nearly crystallized than at present.

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DISCUSSIONS

A PLAN FOR A MOVABLE DAM

Discussion

BY C. L. HALL, C. E. MEYERDICK, AND RALPH R. LEFFLER

C. L. HALL,[2] M. AM. SOC. C. E.[2a]—In his classification of movable dams, Mr. DeYoung includes submerged structures as type (2). Among the submerged structures mentioned are trestle dams, bear traps, Tainter gates, etc., but not wicket dams of the Ohio River type. Wicket dams are found on a great many canalized rivers. In particular there are forty-three such structures on the Ohio River and through them pass the heaviest river-borne commerce in the western hemisphere. Any general discussion of movable dams which does not mention dams of the Ohio River type is likely to give a false impression. Moreover, the classification of a lock gate as a form of movable dam is still more likely to confuse the reader.

The structure described by Mr. DeYoung[2b] is a substitute for the Poiree dam[3] which has been installed in several places to stop downstream flow through a damaged lock, thus permitting repairs to be made. The main difference between the two structures is that the Poiree dam merely provides the horses and props against which wickets may be placed to establish a closure, whereas the flexible shutter type of dam includes substitutes for wickets as integral parts of the dam, thus establishing closure automatically as soon as the dam is raised. Clearly this feature is desirable; but also it is much more expensive. There are comparatively few places where such complex dams are justified. Also, there are only a few locks where a gate failure would be so disastrous that both a submerged movable dam and a set of guard gates would be economically justifiable. For most waterways the extra insurance is not worth the cost.

A study of the drawings indicates that natural sedimentation into the recesses is not feared. In a clear stream like St. Marys River there may be no sedimentation during the long periods when the dam lies on the bottom. In streams that are closed every winter, it may be possible to raise and lower

NOTE.—This paper by Isaac DeYoung was published in February, 1945, *Proceedings*.

[2] Col., Corps of Engrs., U. S. Army, Columbus, Ohio.

[2a] Received by the Secretary March 9, 1945.

[2b] Correction for *Transactions:* In February, 1945, *Proceedings*, page 172, line 2 below Fig. 5, change "Fig. 6" to "Fig. 7."

[3] "The Design and Construction of Dams," by Edward Wegmann, John Wiley & Sons, Inc., New York, N. Y., 8th Ed., p. 309.

the dam every spring before navigation begins; but on the Ohio River this form of dam would probably not operate. In the upper river, acid would probably corrode the steel, unless it were made of a very expensive alloy. Throughout the entire river, sediment would be deposited in any recess, in such a way that it would probably be extremely difficult to overcome initial inertia when dam raising is undertaken. With the river open to navigation all year, it would never be easy to find a convenient time to raise and inspect a flexible shutter type of dam.

The writer expresses no opinion on the proposed dam, considered from the standpoint of the mechanical engineer. He has great doubt as to the practicability of using such a dam except in a canal or canalized river with far from normal characteristics.

C. E. MEYERDICK,[4] ESQ.[4a]—There are various methods of operating locks with guard gates and supplemental gates. The contribution by Mr. DeYoung should be greatly appreciated because it is reasonable that few locks should be built without provision for stopping the flow through the lock chamber in case of an accident. There are, of course, many ideas on how to put a set of lock gates "into the dry"—by constructing bulkheads or by making use of the various forms of movable dams enumerated by Mr. DeYoung. A lock is not necessarily guarded by the use of a set of guard gates as illustrated in Fig. 1.

A simple plan for operating a lock is the plan used on most of the Mississippi River locks from St. Louis, Mo., to St. Paul, Minn. A broken set of gates would tie up traffic through the lock, in which case an auxiliary lock is usually available. Should the lock be left entirely open, the quantity of flow through the lock would not affect other traffic activities on the river since this flow would unquestionably be less than normal flow in the river. If a navigation canal is small in size, its capability to absorb a high flow such as might surge through an open lock chamber is limited. Lock chambers on the Mississippi River are equipped with only a single set of gates at each end of the lock. Normally, one set of gates is always closed. For any other arrangement of gates, if more are considered essential, it does not necessarily follow that expenses should be increased by operating all the gates, as some of them may be left recessed to be used as stand-by units in cases of emergency. The fact that a lock has two sets of gates at each end does not necessitate utilizing both sets to lock a vessel through the river. The cost of these extra gates, including that of the extra set of gates to be used as guard gates, and the cost of movable dams (bulkheads) recessed in the bottom of the lock or otherwise installed add up to a tidy sum for construction—not to mention operation and maintenance.

There are two types of lock gates commonly considered for navigation projects. They are the miter gate and the sector gate. Both types are usually installed in pairs with a sealing arrangement on the gates to effect a seal at the center of the lock where the gates meet, and with other seals at each wall from which each gate swings. The operating procedure for the miter gate is similar to that for the garden gate since the force usually is applied at the top of the gate. In the case of the miter lock gate, considerably more force is

[4] Mech. Engr., U. S. Engr. Office, Sacramento, Calif.

[4a] Received by the Secretary March 19, 1945.

required to swing the gate through the water than is required to swing it free in air.

In sector gate operation, the face plate, being an arc, slices through the water as the gate is swung. The resistance against the sector gate is much less than that against a miter gate unless the gate has a plate on one side in addition to the radial face plate. The operating force on the miter gate commonly is applied through the medium of an operating arm attached to the top of the gate, whereas on the sector gate the force might be applied through a pinion mounted on the lock walk and meshed with a curved rack attached to the top of the face plate of the gate.

The sector gate recess requires more room than does the miter gate recess and both gate types have other advantages and disadvantages. One of the chief advantages of the sector gate is that, because of its slicing action, it can operate against a head differential and in high velocities of flow, whereas operation of the miter gate must never be risked in high velocities. Sector gates in any combination of sets in a lock, by using any one set of the gates, provide a movable dam. The lock chambers can be "valved" by the operation of the sector gates, and there is an indication of the economy of their uses—namely, for "valving," "locking," and emergency operation against high velocities.

The writer favors sector gates because of their sturdiness and minimum deflection in the gate structure during operation. The fact that the resultant of all hydrostatic load lies through the center of the hinge (trunnion) indicates the reason for mechanical economies and utility of operation. A lock can be built with only two sets of sector gates that will meet the necessary requirements of many of the most important navigation structures.

Obviously, as proposed by Mr. DeYoung, the safety of lock operation should extend to closing off the lock in case of accidental damage to any of the locking or valving mechanisms. Mr. DeYoung is to be commended for emphasizing the necessity for safety requirements for navigation structures. The writer believes, however, that by using sector lock gates the margin of safety that can be attained will be comparable to what might be expected by installing a movable dam mechanism as proposed by Mr. DeYoung.

RALPH R. LEFFLER,[5] ESQ.[5a]—The plan outlined in this paper deserves full consideration wherever the problem of designing a movable dam arises. On certain waterways that are important in wartime, such a movable dam should be installed in preparation for World War III. The cost is relatively so small that many major peacetime locks should be equipped with it.

The prospects that Mr. DeYoung's plan will receive merited consideration are not good. The proposal is new; it is novel; it is unprecedented; it is irritating to the "rut-worn" mind; and it has not been done before. It must run the gauntlet of the heavy hands of the finical designers, the high-powered executives, the "solid-minded" citizens, the "buttocky" bureaucrats, the "practical" men, and a whole host of others, adept and zealous at dodging responsibility to safeguard themselves. The design has so many merits, nevertheless, that it will overcome these obstacles.

[5] Engr. of Structural Design, The San. Dist. of Chicago, Chicago, Ill.

[5a] Received by the Secretary April 19, 1945.

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DISCUSSIONS

ELIMINATION OF CROSS CONNECTIONS IN LOS ANGELES, CALIF.

Discussion

BY ROY O. VAN METER

ROY O. VAN METER,[9] ESQ.[9a]—The cross-connection control and elimination program in Los Angeles has been described by Mr. Hayes in a comprehensive manner. The subject is so broad that many interesting problems could be included. The City of Los Angeles did not embark on a survey to enumerate the cross connections but rather to eliminate all cross connections that were found. The surveys were started on a piecemeal basis. A certain critical area was laid out for surveys, a single property in the area was inspected, orders to eliminate the cross connections were issued, and the inspectors diligently prosecuted the case until every cross connection was eliminated or corrected. This procedure sounds easy, but in practice it resulted in many painful experiences.

Mr. Hayes has described the formation of the Los Angeles Cross-Connection Control Advisory Committee and also the problems confronting the committee prior to and after the outbreak of World War II. The permanent chairman of the committee had at his command just three trained engineers on cross-connection control work at the time the Los Angeles Harbor survey was started. The Department of Building and Safety provided six plumbing inspectors untrained in cross-connection control work and later increased the number to eleven, after the first group had become familiar with the work.

To complete the survey as soon as possible and to utilize the trained men to the fullest extent, the work, legal orders, and other features of the survey were standardized as much as possible. In general, the survey of a given area is conducted by the inspector in nine steps as follows: (1) A surface map is prepared; (2) file records are made on service cards; (3) the owner of the area is informed of the problem; (4) an underground location survey is made; (5) a legal order is prepared; (6) the legal order is presented to the owner; (7) any necessary steps are taken to enforce the legal order; (8) corrections

NOTE.—This paper by Harry Hayes was published in March, 1945, *Proceedings*.

[9] Chf. San. Inspector, Bureau of Water Works and Supply, Los Angeles, Calif.

[9a] Received by the Secretary April 16, 1945.

having been made, the area is reinspected; and (9) final inspection having been approved, the case is closed.

(*1*) *The Map.*—A tracing of the area to be surveyed is made from a district sheet provided by the Department of Water and Power. The scale is 100 ft to the inch, and shows the location and size of the street mains; the locations, numbers, sizes, and serial numbers of the water services; the property lines; the street locations; and street names and numbers.

(*2*) *Service Cards.*—Information from service cards is then obtained. Each card (on which are listed meter number and size, owner's name and address, collection name and address, when the meter was set and turned on or off, and other pertinent data) is checked against information obtained in the field.

(*3*) *Informative Visit.*—Contact is made with the owner or manager of the premises and a full and complete detailed explanation is given of the historical background; the prevalence of cross connections, water-borne diseases, water-borne disease epidemics; the formation of the Cross-Connection Control Advisory Committee and its powers and duties; federal, state, and local laws; rules and regulations; enforcement policies; the seven-point program of cross-connection control and elimination; types of cross connections normally found; methods of control and elimination; duties and liabilities of the owners or managers; the type of educational and cooperative campaign the committee is conducting; and the need for making an inspection of premises. After the introductory talk, permission to make the survey is requested; the manager is also asked to furnish a competent engineer or foreman who is familiar with the water system to help on the survey. Blueprints are sometimes found helpful; but, in every case, the information on them must be proved.

(*4*) *Underground Location Survey.*—A physical survey is made of every foot of exposed piping and the underground system is checked as accurately as possible. All lines are sketched on the tracing in the field. A water-use survey is then made to determine whether there are any cross connections or conditions that might tend to pollute the drinking water system.

(*5*) *Preparation of the Legal Order.*—In writing the legal order to eliminate the cross connection, the inspector must present to the assistant engineer in charge of the field work a summary of his findings together with sketches, blueprints, and piping layout. At this conference each cross connection is reviewed and classified as thoroughly as possible, and special cases are treated as independent problems. After deciding on the proper procedure to be followed in each case, the inspector writes his order using a standard wording form that has already been determined. Special wording may be used when existing wording forms do not apply. After the order is written, it is again checked by the assistant engineer. Then it is reviewed by the engineer in charge of field work; and finally the inspector must demonstrate the soundness of the order to the engineer while the two are reviewing the work together on the premises. If the order displays a good sound sanitary engineering basis, it will be approved for service.

(*6*) *Presenting the Legal Order.*—The legal order is then issued to the owner or manager instructing him to eliminate or correct the insanitary conditions. The legal order lists the cross connection or condition; location; proposed

method of elimination; length of time for compliance; instructions as to what to do in case an extension of time is necessary; the laws, rules, or regulations that have been violated; complainants or enforcement agencies involved; and defendants and their duties and obligations during period of compliance.

(7) *Enforcing the Legal Order.*—Pressure is then brought to bear on the owner or operator to gain compliance as soon as possible. This may take the form of a recapitulation of the explanation given at the original conference (step (3)); the presentation of copies of the laws involved; conferences with keymen; letters from the city attorney regarding legal status; letters from the city health officer certifying the need for immediate compliance to help the management in efforts to gain priorities; orders from the Industrial Accident Commission, Naval and Army Security Officers, and other interested agencies; reinspections; etc.

(8) *Final Inspection.*—A final reinspection is made after all the work has been completed.

(9) *Closing the Case.*—The completed orders together with the maps, sketches, field notes, etc., are filed for future use as the War Department requires a continuing inspection of all defense plants.

The cross-connection problem was recognized as a health menace by public health officers several years prior to 1919. In that year, New Hampshire passed a law prohibiting the maintenance of such connections. Between 1920 and 1930, many similar laws were enacted. The problem received official national recognition in 1930, when the surgeon general of the United States issued a directive to all health officers, in which he ordered the outright elimination of cross connections or the installation of "* * * modern protective devices approved by the State Department of Health and installed and operated under the direct control of that Department." Cross connections are not effectively eliminated by the mere passage of laws or by surveys such as the $1,000,000 survey in Chicago, Ill., after the amebic dysentery outbreak in 1933; the $1,081,830 survey conducted in Cleveland, Ohio, in 1942; or the cross-connection survey made by the U. S. Public Health Service and the Works Progress Administration of all federal buildings located in Detroit, Mich., and New York, N. Y., in 1936 and 1937—unless such surveys are followed up by the issuance of individual orders to make necessary corrections; and, what is far more important, these orders must be enforced.

To those who have been endeavoring to control the cross-connection problem, it has become quite obvious that the situation has been growing worse instead of better. This is true for at least seven reasons, as follows: (*a*) Failure to pass control laws, (*b*) failure to enforce laws, (*c*) failure to publicize the problem, (*d*) failure to control the design of water supply systems, (*e*) failure to standardize and codify good practice, (*f*) failure of management to recognize the problem, and (*g*) failure of the engineer to promote good practice.

(*a*) *Control Laws.*—The failure of federal, state, and local lawmakers to pass cross-connection control laws—or rather cross-connection control laws that are uniform, specific, and at the same time understandable—is a contributing cause of the present situation. In most cases, engineers make the

preliminary draft of the laws, after which the draft is referred to the attorneys for a redraft. When the engineers again see the draft, they often cannot recognize it as the product of their efforts. Therefore, they abandon the effort in disgust; the draft is approved by the lawmakers and codified. When the enforcement officer tries to enforce the provisions of the law, he is told by the attorneys that nothing can be done because the law contains too many loopholes and that a conviction could not be secured. In other words, the attorneys would have used the word, "may," where they should have used the word, "shall." Instead of being specific and describing the cross connection involved, the attorneys are vague to the point of ambiguity. Again, the attorneys fail to realize that the proper control of each cross connection is just as important in the problem of health and sanitation as is the determination of an individual stress in the design of a bridge truss.

(*b*) *Law Enforcement.*—The failure on the part of federal, state, and local law enforcement agencies to enforce such laws as are on the books—either because of negligence, ignorance, or fear of political pressure groups—is another contributing cause of present conditions.

(*c*) *Publicity.*—A third cause is the failure on the part of all concerned to publicize the cross-connection control problem. This results from the fear of enforcement officers that they will be criticized because such a condition was allowed to exist, the fear of political reprisals by financially interested pressure groups, and the fear of criminal and civil actions which might be filed against enforcement officers, cities, or utilities (both public and private) for past failure to perform duties and obligations properly. The cross-connection control problem is just as widespread and just as important as the syphilis problem, for example, and the sooner the public recognizes that fact and publicizes the cause and the cure, the sooner the situation can be controlled.

(*d*) *Design Controls.*—Federal, state, and local health authorities fail to recognize the need for specific regulations governing the design of water piping systems, the use of water in all its phases, and the continuing inspection of water systems and water uses. Little, if any, effort has been made to prevent either the improper design of water systems or the overloading of existing systems. The overloading of existing water systems presents one of the greatests hazards in cross-connection control. In one plant a sustained vacuum of 16 in. of mercury was recorded for a period of 5 min in the water system on the second floor. The operating head during the daytime was 5 lb per sq in. in the system on the first floor.

(*e*) *Good Standard Practice.*—Federal and state public health authorities fail to set up a "uniform practice code" for the elimination of cross connections, the testing of backflow protective devices, and the certifying of modern backflow protective devices. Everyone, without regard to authority or fitness, seems to have a voice in making decisions, and local health enforcement agencies are criticized and crucified if they propose any advanced ideas. In most cases the only approved method is the obsolete double check-valve assembly proposed about 1925. The principle of installing vacuum breakers and pressure differential releases on every pressure-operated backflow protective device should be universally adopted. Inventors, designers, manufacturers, and others are

afraid to spend their money to develop new ideas for modern backflow protective devices because they know they will not be able to recover their money or because they fear that they will be accused of promoting a device on which they have a monopoly. If the standards were made high enough, and were enforced, many devices would be developed. There is a growing demand for dependable equipment.

(*f*) *Responsibility of Management.*—Management fails to give due consideration to the cross-connection control problem, and is often willing to expend large sums of money for the installation of water systems without consulting local authorities regarding sanitary engineering requirements. The fault for this condition lies with the enforcement authorities rather than with the plant management. The authorities have assumed that the management has as much knowledge of the subject as experts. This is especially evident in the large industrial plants where extensive changes have been ordered. After the reasons for the required changes have been explained, the management has usually concurred and has cheerfully ordered the work completed. In several cases the work order has involved the expenditure of more than $50,000 for a single area.

(*g*) *Responsibility of Engineers.*—Engineers, individually and collectively in their associations, fail to give more than lip service toward solving the problem. The individual plant engineer tries to establish a record for economy. Either consulting and designing engineers are not familiar with water piping and cross-connection problems, or they deliberately fail to incorporate in designs the necessary safety factors required to insure a safe and potable water supply. The associations have made studies and recommendations; but, as yet, they have not made insistent efforts to see that the results of their studies are incorporated in a "Uniform Water Piping and Cross-Connection Control Code."

Summary.—So far as cross-connection control is concerned, enforcement officers have conspicuously failed in carrying out their sworn duty—that is, the safeguarding of public health; the formulation of equitable laws; and, what is far more important, the enforcement of all laws in a uniform manner without fear of political pressure groups and without favor to minority groups.

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DISCUSSIONS

ORGANIZATION, FINANCING, AND ADMINISTRATION OF SANITARY DISTRICTS

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION

Discussion

BY C. MAXWELL STANLEY

C. MAXWELL STANLEY,[2] M. AM. SOC. C. E.[2a]—This report is timely because of the increasing need for sanitary districts to collect and treat sewage where the logical scope of a system encompasses areas extending beyond the corporate limits of a municipality. Many states, without enabling legislation, will require suitable statutes in the near future. The committee report will be helpful in guiding the preparation of such statutes and the establishment of sanitary districts.

Part I. Organization.—The outline of fundamental considerations is well stated. The writer concurs with the committee's suggestion and favors the creation and organization of sanitary districts by local petitions. It is advisable that the largest practicable degree of local initiative be permitted in the organization of districts.

It should be emphasized that the determination of the extent of the sanitary district is primarily an engineering problem, not a political one. Provisions must be made for adequate funds to permit preliminary engineering surveys and investigations prior to defining the area to be included. Such studies will more than pay for themselves by assuring logical development.

The selection of the method to be used in setting up the administrative board should be based, to a considerable extent, upon local practice and experience. For instance, in Iowa, trustee management has proved very satisfactory for municipal utilities. The trustees (usually three in number) are appointed by the mayor and approved by the council for terms of six

NOTE.—This report was published in February, 1945, *Proceedings*.

[2] Cons. Engr. (Stanley Eng. Co.), Muscatine, Iowa.

[2a] Received by the Secretary April 4, 1945.

years, with the expiration of terms staggered so that one appointment is made every two years. This feature provides a highly desirable continuity of management. In other areas, different methods of administrative management may have proved more successful and may properly be selected for sanitary districts. The committee's suggestions on size of board and salary appear to be sound.

The necessity of making provisions for preliminary financing of a district should be emphasized. The enabling legislation should definitely provide for such preliminary financing in order that proper engineering and legal work may be done before the district boundaries are determined and the district is formally established. The power to issue securities and levy taxes should be added to the list of the functions of a district.

Part II. Finance.—The problems of financing a sanitary district are not appreciably different from those encountered in financing any sewerage project. The scope of the project to be financed will vary with local conditions, but undoubtedly most sanitary districts will be primarily concerned with interceptor sewers and sewage treatment plants, rather than with the collection of sewage. Normally, the incentive that leads to the formation of a district will result from such sewage problems.

It is important that the selection of the method of financing be made only after consideration of prior practices. For instance, if the established practice in the community has been to finance lateral sewers by special assessment, a program to finance additional lateral sewers by general obligation bonds, paid by general taxes, will be unfair to those property owners who have paid for their lateral sewers by special assessment.

In considering the over-all financing of sewerage projects, a combination of the three methods involving general taxes, assessments, and rentals would very frequently be advisable. In planning such a combination, every effort must be made to avoid undue complications. Thus, frequently, some minor degree of equity must be sacrificed for simplicity and workability.

The writer concurs in the committee's suggestion that sanitary districts be provided with broad rights of financing permitting them to select a program that will assure equality and will still be flexible enough to meet changing conditions. All problems cannot be foreseen at the time enabling legislation is enacted, and the granting of broad financing powers will avoid future complications.

It has been the writer's observation that, in a great many cases, the collection of sewage rentals by an addition to water rates is a most suitable procedure. Enabling legislation should permit sanitary districts to cooperate with water works administrations for the establishment and collection of rentals on this basis.

The writer is not prepared to comment on the legal problems involved in the discussion of the governmental and proprietary aspects of sewerage projects. From an engineer's point of view, however, it seems that there are sufficient elements of both functions present to justify combinations in the methods of financing. Public health is generally conceded to be a governmental function.

Certainly, then, some aspects of sewage projects must be governmental; but water supply, although generally conceded to be proprietary, also concerns public health. It is possible to separate governmental and proprietary functions. Governmental agencies may establish the regulations deemed necessary in the interest of public health. The service itself may then be provided by private ownership. This is actually done in some cases for water supply. Theoretically, it is possible to do the same in the sewage field. The state, and subdivisions of the state, may exercise governmental functions by establishing standards and requirements that must be met to protect public health. The actual service could then be provided by private capital. Actually, this is not practicable because the lack of opportunity for financial profit will not attract private capital. Therefore, municipalities and sanitary districts must provide the service, and in so doing they seem to be exercising a proprietary function.

Corrections for *Transactions:* In February, 1945, *Proceedings*, page 185, change the description of Item 12 to read, "Number of consumers in minimum bracket," and change the next line following it to read, "Number of Intermediate Users"; also, on page 187, in bibliography reference 4, change "Vol. 105" to "Vol. 109."

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DISCUSSIONS

CONVERSION OF KINETIC TO POTENTIAL ENERGY IN FLOW EXPANSIONS

Discussion

BY J. C. STEVENS

J. C. STEVENS,[29] PRESIDENT, AM. SOC. C. E.[29a]—Some of the mental confusion that has surrounded the particular problem of converting kinetic energy to potential energy in flow expansions has been dispelled by Professor Kalinske. He has shown that turbulence, heretofore looked upon as the principal source of ultimate energy loss, is merely a by-product of the direct loss (energy conversion into heat) in the zone of shear surrounding the jet.

The author found that the maximum energy of turbulence was only half the maximum energy of the mean flow; and, of course, only a part of this turbulent energy is converted into heat as the intensity diminishes downstream. For example, in the 30° expansion, the total energy lost was 4.3 ft-lb per sec for 0.210 cu ft per sec (Fig. 6); and the maximum energy of turbulence was 0.781 ft-lb per sec (Table 2). Even if all this turbulence were lost (converted into heat), it would represent only 18% of the total energy similarly lost in the expansion.

The efficiency of an expansion is subject to various interpretations. The author has used the ratio of the gain in potential energy to the loss in kinetic energy (both in foot-pounds per second) as a measure of the efficiency of the expansion. This is primarily a measure of the recovery of velocity head. It seems to the writer that efficiency is something more than this. If the pipe length from the beginning of the expansion to the point where normal flow is restored in the larger pipe (say, sixteen diameters) is considered as a mechanism for converting from one state of flow to another, the efficiency becomes the ratio of the output energy to the input energy as is the case with any other device.

Curiosity caused the writer to prepare Table 6, comparing the orthodox concepts of efficiencies with those shown by the author in Table 4. The input and

NOTE.—This paper by A. A. Kalinske was published in December, 1944, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1945, by F. T. Mavis.

[29] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[29a] Received by the Secretary March 16, 1945.

output energies were taken from Fig. 6. In either expansion, and by either method, the lower the flow, the higher the efficiency would be—approaching 100% for zero flow.

The tedium of analyzing so many movie-camera frames prevented the extension of the experiments to 60° expansions and 90° expansions. It would be interesting to know what rate of expansion yields the highest rate of energy loss—in other words, the maximum scale of turbulence.

TABLE 6.—COMPARISON OF EFFICIENCIES

Flow (cu ft per sec)	7.5° EXPANSION				30° EXPANSION				180° EXPANSION			
	Input, Station 0 ft $\left(\frac{E_{m1}}{E_{p1}}\right)$	Output, Station 6.5 ft $\left(\frac{E_{m2}}{E_{p2}}\right)$	EFFICIENCY (%)		Input, Station 0 ft $\left(\frac{E_{m1}}{E_{p1}}\right)$	Output, Station 6.5 ft $\left(\frac{E_{m2}}{E_{p2}}\right)$	EFFICIENCY (%)		Input, Station 0 ft $\left(\frac{E_{m1}}{E_{p1}}\right)$	Output, Station 6.5 ft $\left(\frac{E_{m2}}{E_{p2}}\right)$	EFFICIENCY (%)	
			Orthodox (η')	Table 4 (η)			Orthodox (η')	Table 4 (η)			Orthodox (η')	Table 4 (η)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0.082	..	..	..	..	..	..	..	..	0.78	0.63	81	51
0.147	2.8	2.4	85	81	2.8	1.9	68	59	2.8	1.7	61	45
0.210	8.2	5.1	62	60	8.2	4.3	52	48	8.0	4.0	50	43

Professor Kalinske used the only method so far devised, of which the writer is aware, for measuring the components of filament velocities. It is hoped some method will be devised whereby such components can be measured with less time-consuming labor. Perhaps the method evolved at Columbia University,[30] in New York, N. Y., might be extended to the measurement of filament velocities in pipes of nonmagnetic material.

Research into these fundamental concepts should not end with this paper; this study is just a good beginning. A most excellent approach to determining the nature of hydraulic friction is that presented by Boris A. Bakhmeteff, M. Am. Soc. C. E., and William Allan, Assoc. M. Am. Soc. C. E., in February, 1945.[31] The findings in the researches by Professor Kalinske and by Professors Bakhmeteff and Allan dovetail most admirably and open new vistas for the hydraulician.

As noted by the author (see heading, "Acknowledgments"), the project reported in the paper was sponsored by the Hydraulic Research Committee of the Society which, in 1938, became the Committee on Hydraulic Research of the Hydraulics Division. The Committee is convinced that little further progress may be expected in the solution of many hydraulic problems until the internal mechanics of hydraulic friction and energy conversions in flowing water are better known. In this respect the committee is justly proud of the work Professor Kalinske has done and of the results he has obtained.

[30] "Electromagnetic Velomentry I. A Method for the Determination of Fluid Velocity Distribution in Space and Time," by Alexander Kolin, *Journal of Applied Physics*, Vol. 15, No. 2, February, 1944, pp. 150–164.

[31] "The Mechanism of Energy Loss in Fluid Friction," by Boris A. Bakhmeteff and William Allan, *Proceedings*, Am. Soc. C. E., February, 1945, p. 129.